Matemáticas I 1º Bachillerato

HOJA 1 DE EJERCICIOS
UNIDAD 1: TRIGONOMETRÍA I

Ejercicio 1: Dados los ángulos, $\alpha = 35^o46'52''$, $\beta = 46^o53'18''$, $\omega = -20^o11'23.5''$ y $\gamma = 142^o53'1''$ efectúa las siguientes operaciones con ángulos sexagesimales:

a) $\alpha+\beta-\omega$ b) $\alpha-\beta$ c) $3\cdot\omega$ d) $\frac{1}{3}\cdot\beta$ e) $\frac{2}{5}\cdot\gamma-\alpha$

a) $102^o 51' 33.5''$

d) $15^o 37' 46''$

b) $-11^o 6' 26''$

e) $21^o 22' 20.4''$

c) $-60^o 34' 10.5''$

Ejercicio2: Pasa a grados sexagesimales los siguientes ángulos en radianes:

a) $\frac{\pi}{12}$ b) $\frac{17\pi}{6}$ c) 2

a) 15^o b) 510^o c) $114^o 35' 29.6''$

Ejercicio 3: Pasa a radianes los siguientes ángulos:

a) 75^o b) -195^o c) $22^o30'$ d) 370^o

a) $\frac{5\pi}{12}$ rad b) $-\frac{13\cdot\pi}{12}$ rad c) $\frac{\pi}{8}$ rad d) $\frac{37\pi}{18}$ rad

Ejercicio 4: Sabiendo que $\cos\alpha = \frac{1}{4}$, y que $270^o < \alpha < 360^o$, calcula las restantes razones trigonométricas del ángulo α.

$\cos\alpha = \frac{1}{4}$ $\quad \text{sen}\,\alpha = -\frac{\sqrt{15}}{4}$ $\quad tg\,\alpha = -\sqrt{15}$

$\sec\alpha = 4$ $\quad \text{cosec}\,\alpha = -\frac{4\sqrt{15}}{15}$ $\quad \text{cotg}\,\alpha = -\frac{\sqrt{15}}{15}$

Ejercicio 5: Calcula las razones trigonométricas en los siguientes casos:

a) $sen\,\alpha = \frac{1}{4}$ y $\alpha \in I$ Cuadrante

$\text{sen}\,\alpha = \frac{1}{4}$ $\quad \cos\alpha = \frac{\sqrt{15}}{4}$ $\quad tg\,\alpha = \frac{\sqrt{15}}{15}$

$\text{cosec}\,\alpha = 4$ $\quad \sec\alpha = \frac{4\sqrt{15}}{15}$ $\quad \text{cotg}\,\alpha = \sqrt{15}$

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b) $\cos\alpha = \dfrac{2\sqrt{3}}{5}$ y $\alpha \in IV$ Cuadrante

$\cos\alpha = \dfrac{2\sqrt{3}}{5}$ $\quad$ $\text{sen}\,\alpha = \dfrac{-\sqrt{13}}{5}$ $\quad$ $tg\,\alpha = -\dfrac{\sqrt{39}}{6}$

$\sec\alpha = \dfrac{5\sqrt{3}}{6}$ $\quad$ $\text{cosec}\,\alpha = -\dfrac{5\sqrt{13}}{13}$ $\quad$ $\text{cotg}\,\alpha = \dfrac{-6\sqrt{39}}{39}$

c) $tg\,\alpha = 2$ y $\alpha > 90^\circ$

Caso 1: $\alpha > 90^\circ$ y $\alpha \in III$ Cuadrante

$\cos\alpha = -\dfrac{\sqrt{5}}{5}$ $\quad$ $\text{sen}\,\alpha = -\dfrac{2\sqrt{5}}{5}$ $\quad$ $tg\,\alpha = 2$

$\sec\alpha = -\sqrt{5}$ $\quad$ $\text{cosec}\,\alpha = -\dfrac{\sqrt{5}}{2}$ $\quad$ $\text{cotg}\,\alpha = 2$

Caso 2: $\alpha > 90^\circ$ y $\alpha > 360^\circ$ del I Cuadrante

$\cos\alpha = \dfrac{\sqrt{5}}{5}$ $\quad$ $\text{sen}\,\alpha = +\dfrac{2\sqrt{5}}{5}$ $\quad$ $tg\,\alpha = 2$

$\sec\alpha = \sqrt{5}$ $\quad$ $\text{cosec}\,\alpha = \dfrac{\sqrt{5}}{2}$ $\quad$ $\text{cotg}\,\alpha = \dfrac{1}{2}$

d) $\text{sen}\,\alpha + \cos\alpha = \sqrt{2}$ y $\alpha \in I$ Cuadrante

$\cos\alpha = \dfrac{\sqrt{2}}{2}$ $\quad$ $\text{sen}\,\alpha = \dfrac{\sqrt{2}}{2}$ $\quad$ $tg\,\alpha = 1$

$\sec\alpha = \sqrt{2}$ $\quad$ $\text{cosec}\,\alpha = \sqrt{2}$ $\quad$ $\text{cotg}\,\alpha = 1$

e) $\sec\alpha = -3$ y $\alpha \in III$ Cuadrante

$\cos\alpha = -\dfrac{1}{3}$ $\quad$ $\text{sen}\,\alpha = \dfrac{-2\sqrt{2}}{3}$ $\quad$ $tg\,\alpha = 2\sqrt{2}$

$\sec\alpha = -3$ $\quad$ $\text{cosec}\,\alpha = \dfrac{-3\sqrt{2}}{4}$ $\quad$ $\text{cotg}\,\alpha = \dfrac{\sqrt{2}}{4}$

Ejercicio 6: Calcula las siguientes razones trigonométricas sin usar la calculadora:

a) $\text{sen}\,240^\circ$ $\quad$ b) $tg\,120^\circ$ $\quad$ c) $\text{sen}\,\dfrac{3\pi}{4}$ $\quad$ d) $\cos\dfrac{5\pi}{3}$ $\quad$ e) $tg\,750^\circ$

f) $tg\,(-30^\circ)$ $\quad$ g) $\sec\left(-\dfrac{5\pi}{4}\right)$ h) $\text{cotg}\,\dfrac{37\pi}{6}$ $\quad$ i) $\text{cosec}\,585^\circ$

a) $\text{sen}\,240^\circ = -\text{sen}\,60^\circ = -\dfrac{\sqrt{3}}{2}$ $\quad$ b) $tg\,120^\circ = -tg\,60^\circ = -\sqrt{3}$

c) $\text{sen}\,\dfrac{3\pi}{4} = \text{sen}\,\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$ $\quad$ d) $\cos\dfrac{5\pi}{3} = \cos\dfrac{\pi}{3} = \dfrac{1}{2}$

e) $tg\,750^\circ = tg\,30^\circ = \dfrac{\sqrt{3}}{3}$ $\quad$ f) $tg\,(-30^\circ) = -tg\,30^\circ = -\dfrac{\sqrt{3}}{3}$

g) $\sec\left(-\dfrac{5\pi}{4}\right) = \dfrac{1}{\cos\left(-\dfrac{5\pi}{4}\right)} = \dfrac{1}{\cos\dfrac{5\pi}{4}} = \dfrac{1}{-\cos\dfrac{\pi}{4}} = -\sqrt{2}$

h) $\cot g \frac{37\pi}{6} = \frac{1}{tg\frac{37\pi}{6}} = \frac{1}{tg\frac{\pi}{6}} = \frac{1}{\frac{\sqrt{3}}{3}} = \sqrt{3}$

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Ejercicio 7: Si $\text{tg}\ \alpha = \frac{2}{3}$ y $0° < \alpha < 90°$, halla: $\Rightarrow \cos\alpha = \frac{3\sqrt{13}}{13}$ $\text{sen}\alpha = \frac{2\sqrt{13}}{13}$

a) $\text{sen}\ \alpha$ b) $\cos\ \alpha$ c) $\text{tg}\ (90° - \alpha)$

d) $\cos\ (180° + \alpha)$ e) $\text{sen}\ (180° - \alpha)$ f) $\text{tg}\ (360° - \alpha)$

a) $\text{sen}\alpha = \frac{2\sqrt{13}}{13}$ b) $\cos\alpha = \frac{3\sqrt{13}}{13}$ c) $tg(90° - \alpha) = \frac{3}{2}$

d) $\cos(180° + \alpha) = -\cos\alpha = -\frac{3\sqrt{13}}{13}$ e) $\text{sen}(180° - \alpha) = \text{sen}\,\alpha = \frac{2\sqrt{13}}{13}$

f) $tg(360° - \alpha) = -tg\,\alpha = -\frac{2}{3}$

Ejercicio 8: Sabiendo que $sen\ \alpha = \frac{-3}{5}$, y que $\alpha \in III$ Cuadrante, calcula: $\cos\alpha = -\frac{4}{5}$

a) $\text{tg}\ \alpha$ b) $\cos\left(\frac{\pi}{2} - \alpha\right)$ c) $\text{sen}\ (\pi + \alpha)$ d) $\sec\ (180° - \alpha)$

e) $\text{cotg}\ (-\alpha)$ f) $\text{sen}\ (8\pi + \alpha)$ g) $\text{cosec}\left(\frac{\pi}{2} + \alpha\right)$ h) $\text{sen}^2\ (\pi + \alpha) + \cos^2\left(\frac{\pi}{2} - \alpha\right)$

a) $tg\,\alpha = \frac{3}{4}$ b) $\cos\left(\frac{\pi}{2} - \alpha\right) = \text{sen}\,\alpha = -\frac{3}{5}$ c) $\text{sen}(\pi + \alpha) = -\text{sen}\alpha = \frac{3}{5}$

d) $\sec(180° - \alpha) = \frac{1}{\cos(180° - \alpha)} = \frac{1}{-\cos\alpha} = \frac{5}{4}$

e) $\cot g(-\alpha) = \frac{1}{tg(-\alpha)} = \frac{1}{-tg\alpha} = -\frac{4}{3}$ f) $\text{sen}(8\pi + \alpha) = \text{sen}\,\alpha = -\frac{3}{5}$

g) $\text{cosec}\left(\frac{\pi}{2} + \alpha\right) = \frac{1}{\text{sen}\left(\frac{\pi}{2} + \alpha\right)} = \frac{1}{\text{sen}\left(\frac{\pi}{2} - \alpha\right)} = \frac{1}{\cos\alpha} = -\frac{5}{4}$

h) $\text{sen}^2(\pi + \alpha) + \cos^2\left(\frac{\pi}{2} - \alpha\right) = (-\text{sen}\,\alpha)^2 + (\text{sen}\,\alpha)^2 = \frac{9}{25} + \frac{9}{25} = \frac{18}{25}$

Ejercicio 9: Comprueba las siguientes identidades trigonométricas:

a) $\text{tg}\ \alpha + \text{cotg}\ \alpha = \sec\ \alpha \cdot \text{cosec}\ \alpha$ b) $\text{cotg}^2\ \alpha = \cos^2\ \alpha + (\text{cotg}\ \alpha \cdot \cos\ \alpha)^2$

c) $\frac{1 - \text{sen}\ \alpha}{\cos\ \alpha} = \frac{\cos\ \alpha}{1 + \text{sen}\ \alpha}$ d) $\frac{\text{cotg}\ \alpha + \text{sen}\ \alpha}{\text{tg}\ \alpha + \text{cosec}\ \alpha} = \cos\ \alpha$

a) $tg\,\alpha + \cot g\,\alpha = \frac{\text{sen}\alpha}{\cos\alpha} + \frac{\cos\alpha}{\text{sen}\alpha} = \frac{\text{sen}^2\alpha + \cos^2\alpha}{\cos\alpha \cdot \text{sen}\,\alpha} = \frac{1}{\cos\alpha \cdot \text{sen}\alpha} = \frac{1}{\cos\alpha} \cdot \frac{1}{\text{sen}\alpha} = \sec\alpha \cdot \text{cosec}\,\alpha.$

b) $\cos^2\alpha + (\cot g\,\alpha \cdot \cos\alpha)^2 = \cos^2\alpha + \frac{\cos^2\alpha}{\operatorname{sen}^2\alpha}\cdot\cos^2\alpha =$

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$= \cos^2\alpha\cdot\left(1+\frac{\cos^2\alpha}{\operatorname{sen}^2\alpha}\right) = \cos^2\alpha\cdot\left(\frac{\operatorname{sen}^2\alpha+\cos^2\alpha}{\operatorname{sen}^2\alpha}\right) = \frac{\cos^2\alpha}{\operatorname{sen}^2\alpha} = \cot g^2\alpha$

c) $\frac{1-\operatorname{sen}\alpha}{\cos\alpha} = \frac{\cos\alpha}{1+\operatorname{sen}\alpha} \iff (1+\operatorname{sen}\alpha)\cdot(1-\operatorname{sen}\alpha) = \cos^2\alpha \iff 1-\operatorname{sen}^2\alpha = \cos^2\alpha$
lo cual es cierto

d) $\frac{\cot g\,\alpha + \operatorname{sen}\alpha}{tg\,\alpha + \operatorname{cosec}\alpha} = \frac{\frac{\cos\alpha}{\operatorname{sen}\alpha}+\operatorname{sen}\alpha}{\frac{\operatorname{sen}\alpha}{\cos\alpha}+\frac{1}{\operatorname{sen}\alpha}} = \frac{\frac{\cancel{\cos\alpha+\operatorname{sen}^2\alpha}}{\cancel{\operatorname{sen}\alpha}}}{\frac{\cancel{\operatorname{sen}^2\alpha+\cos\alpha}}{\cos\alpha\cdot\cancel{\operatorname{sen}\alpha}}} = \cos\alpha.$

Ejercicio 10: Demuestra las siguientes igualdades o identidades trigonométricas:

a) $\cos^2\alpha\cdot\cos^2\beta - \operatorname{sen}^2\alpha\cdot\operatorname{sen}^2\beta = \cos^2\alpha - \operatorname{sen}^2\beta$

b) $\operatorname{cotg}^2\alpha = \cos^2\alpha + (\operatorname{cotg}\alpha\cdot\cos\alpha)^2$

c) $\frac{1-\operatorname{sen}\alpha}{\cos\alpha} = \frac{\cos\alpha}{1+\operatorname{sen}\alpha}$

d) $\frac{\operatorname{cotg}\alpha+\operatorname{sen}\alpha}{\operatorname{tg}\alpha+\operatorname{cosec}\alpha} = \cos\alpha$

a) $\cos^2\alpha\cdot(1-\operatorname{sen}^2\beta) - \operatorname{sen}^2\alpha\cdot\operatorname{sen}^2\beta = \cos^2\alpha - \underline{\cos^2\alpha\cdot\operatorname{sen}^2\beta} - \operatorname{sen}^2\alpha\cdot\underline{\operatorname{sen}^2\beta}$

$= \cos^2\alpha - \operatorname{sen}^2\beta\cdot(\underbrace{\cos^2\alpha+\operatorname{sen}^2\alpha}_{=1}) = \cos^2\alpha - \operatorname{sen}^2\beta$

b) $\cos^2\alpha + \left(\frac{\cos\alpha}{\operatorname{sen}\alpha}\cdot\cos\alpha\right)^2 = \cos^2\alpha + \left(\frac{\cos^2\alpha}{\operatorname{sen}\alpha}\right)^2 = \cos^2\alpha + \frac{\cos^4\alpha}{\operatorname{sen}^2\alpha} = \cos^2\alpha\cdot\left(1+\frac{\cos^2\alpha}{\operatorname{sen}^2\alpha}\right)$

$= \cos^2\alpha\cdot\left(\frac{\operatorname{sen}^2\alpha+\cos^2\alpha}{\operatorname{sen}^2\alpha}\right) = \cos^2\alpha\cdot\frac{1}{\operatorname{sen}^2\alpha} = \frac{\cos^2\alpha}{\operatorname{sen}^2\alpha} = \cot g^2\alpha$

c) Análogo al c) del ejercicio 9

d) $\frac{\frac{\cos\alpha}{\operatorname{sen}\alpha}+\operatorname{sen}\alpha}{\frac{\operatorname{sen}\alpha}{\cos\alpha}+\frac{1}{\operatorname{sen}\alpha}} = \frac{\frac{\cancel{\cos\alpha+\operatorname{sen}^2\alpha}}{\cancel{\operatorname{sen}\alpha}}}{\frac{\cancel{\operatorname{sen}^2\alpha+\cos\alpha}}{\cancel{\operatorname{sen}\alpha}\cdot\cos\alpha}} = \frac{1}{\frac{1}{\cos\alpha}} = \cos\alpha.$

Ejercicio 11: (Uso de la calculadora) Obtén los ángulos siguientes, dando el resultado en grados sexagesimales y en radianes:

a) $sen\ \alpha = \frac{-3}{5}$ con $\alpha \in$ IV Cuadrante

b) $\cos\beta = 0'9659$ con $\beta \in I$ Cuadrante

c) $\operatorname{tg}\gamma = -0'25$ con $\gamma \in$ II Cuadrante

d) $\operatorname{tg}\omega = 0'25$ con $\omega \in$ III Cuadrante

a) $\alpha = -36^\circ\ 52'\ 11.6''$ ó $323^\circ\ 7'\ 48.4''$
$\alpha = -0'6435$ rad ó $5'6397$ rad

b) $\beta = 15^\circ\ 0'\ 20.58''$
$\beta = 0'2619$ rad

c) $\gamma = 165^\circ\ 57'\ 49.5''$
$\gamma = 2'8966$ rad

d) $\omega = 194^\circ\ 2'\ 10.48''$
$\omega = 3'3866$ rad.