EJERCICIOS RESUELTOS DE FRACCIONES ALGEBRAICAS

1. Simplificar las fracciones

a) $\dfrac{x^3+x}{x^4-1}$ b) $\dfrac{m^2-9}{9m-m^3}$ c) $\dfrac{ax+by}{ax^2+bxy}$ d) $\dfrac{x^2-9x}{x^3-6x^2+9x}$ e) $\dfrac{x^4+2x^3-3x^2}{x^4+2x^3+2x^2+10x+15}$

a) $\dfrac{x^3+x}{x^4-1} \Rightarrow \dfrac{x^3+x}{x^4-1}=\dfrac{(x)\cancel{(x^2+1)}}{(x+1)(x-1)\cancel{(x^2+1)}} \Rightarrow \dfrac{x^3+x}{x^4-1}=\dfrac{x}{(x+1)(x-1)}$

b) $\dfrac{m^2-9}{9m-m^3} \Rightarrow \dfrac{m^2-9}{9m-m^3}=\dfrac{\cancel{(m+3)}(m-3)}{m\cancel{(3+m)}(3-m)} \Rightarrow \dfrac{-(m-3)}{-m(3-m)}=\dfrac{\cancel{(-m+3)}}{-m\cancel{(3-m)}} \Rightarrow -\dfrac{1}{m}$

c) $\dfrac{ax+by}{ax^2+bxy} \Rightarrow \dfrac{ax+by}{ax^2+bxy}=\dfrac{\cancel{(ax+by)}}{x\cancel{(ax+by)}} \Rightarrow \dfrac{ax+by}{ax^2+bxy}=\dfrac{1}{x}$

d) $\dfrac{x^2-9x}{x^3-6x^2+9x} \Rightarrow \dfrac{x^2-9x}{x^3-6x^2+9x}=\dfrac{x(x+3)(x-3)}{x(x-3)^2} \Rightarrow \dfrac{x^2-9x}{x^3-6x^2+9x}=\dfrac{x+3}{x-3}$

e) $\dfrac{x^4+2x^3-3x^2}{x^4+2x^3+2x^2+10x-15} \Rightarrow \dfrac{x^4+2x^3-3x^2}{x^4+2x^3+2x^2+10x-15}=\dfrac{x^2(x-1)(x+3)}{(x+3)(x-1)(x^2+5)} \Rightarrow \dfrac{x^2}{(x^2+5)}$

2. Sumar las fracciones

a) $\frac{3}{x-1}+\frac{x}{x+1}+\frac{4}{x^2+1}$ b) $\frac{x-1}{x+2}+\frac{3}{x-2}-\frac{3x+4}{(x+2)^2}-\frac{x+2}{x^2-4}$

a) $\frac{3}{x-1}+\frac{x}{x+1}+\frac{4}{x^2+1}=\frac{3(x+1)(x^2+1)+x(x-1)(x^2+1)+4(x-1)(x+1)}{(x-1)(x+1)(x^2+1)}\Rightarrow$

$$\frac{3x^3+3x^2+3x+3+x^4-x^3+x^2-x+4x^2-4}{(x^2-1)(x^2+1)}=\frac{x^4+2x^3+8x^2+2x-1}{x^4-1}$$

b) $\frac{x-1}{x+2}+\frac{3}{x-2}-\frac{3x+4}{(x+2)^2}=\frac{(x-1)(x+2)(x-2)+3(x+2)^2-(3x+4)(x-2)}{(x+2)^2(x-2)}\Rightarrow$

$$\frac{x^3-x^2-4x+4+3x^2+12x+12-(3x^2-6x+4x-8)}{(x+2)^2(x-2)}=\frac{x^3-x^2+10x+24}{x^3+2x^2-4x-8}$$

3. Efectuar estas operaciones :

a) $\frac{x^2-2x+3}{x-2}\times\frac{2x+3}{x+5}$ b) $\frac{x+2}{x}:\left(\frac{x-1}{3}\times\frac{x}{2x+1}\right)$

a) $\frac{x^2-2x+3}{x-2}\cdot\frac{2x+3}{x+5}=\frac{(x^2-2x+3)(2x+3)}{(x-2)(x+5)}\Rightarrow\frac{2x^3+3x^2-4x^2-6x+6x+9}{x^2+5x-2x-10}=\frac{2x^3-x^2+9}{x^2+3x-10}$

b) $\frac{x+2}{x}:\left(\frac{x-1}{3}\cdot\frac{x}{2x+1}\right)=\frac{x+2}{x}:\left(\frac{(x-1)x}{3(2x+1)}\right)\Rightarrow\frac{x+2}{x}\cdot\frac{3(2x+1)}{(x-1)x}=\frac{6x^2+15x+6}{x^3-x^2}$

4. Efectuar y simplificar el resultado de :

a) $\frac{1}{x+1}\left(x-\frac{1}{x}\right)$ b) $\left(x-\frac{4}{x}\right):(x+2)$

a) $\frac{1}{x+1}\left(x-\frac{1}{x}\right)=\frac{1}{x+1}\left(\frac{x^2-1}{x}\right)\Rightarrow\frac{(x^2-1)}{(x+1)x}=\frac{(x+1)(x-1)}{(x+1)x}\Rightarrow\frac{1}{x+1}\left(x-\frac{1}{x}\right)=\frac{x-1}{x}$

b) $\left(x-\frac{4}{x}\right):(x+2)=\frac{x^2-4}{x}:(x+2)\Rightarrow\frac{(x^2-4)}{x}\cdot\frac{1}{(x+2)}=\frac{(x+2)(x-2)}{x(x+2)}\Rightarrow\frac{x-2}{x}$

5.- Efectuar:

$$\frac{1}{x+1}+\frac{2x}{x^2-1}-\frac{1}{x-1}=$$

$$x^2-1=(x+1)\cdot(x-1)$$

$$\text{m.c.m.}\left(x+1,\ x^2-1,\ x-1\right)=(x+1)\cdot(x-1)$$

$$=\frac{x-1+2x-(x+1)}{(x+1)\cdot(x-1)}=\frac{x-1+2x-x-1}{(x+1)\cdot(x-1)}=\frac{2x-2}{(x+1)\cdot(x-1)}=\frac{2\cdot(x-1)}{(x+1)\cdot(x-1)}=\frac{2}{(x+1)}$$

6.- Efectuar:

$$\frac{x^3+3x^2-4x-12}{x^2+2x-3}:\frac{4x-2x^2}{x^3-2x^2+x}=\frac{(x^3+3x^2-4x-12)\cdot(x^3-2x^2+x)}{(x^2+2x-3)\cdot(4x-2x^2)}=$$

$$=\frac{(x-2)\cdot(x+2)\cdot(x+3)\cdot x\cdot(x-1)^2}{(x+3)\cdot(x-1)\cdot 2x\cdot(2-x)}=\frac{-(x-2)\cdot(x+2)\cdot(x-1)}{2\cdot(-2+x)}=-\frac{(x+2)\cdot(x-1)}{2}$$

7.- Operar:

$$\left(x+\frac{x}{x-1}\right)\cdot\left(x-\frac{x}{x-1}\right)=x^2-\left(\frac{x}{x-1}\right)^2=x^2-\frac{x^2}{(x-1)^2}=$$

$$=\frac{x^2\cdot(x-1)^2-x^2}{(x-1)^2}=\frac{x^2\left[(x-1)^2-1\right]}{(x-1)^2}=\frac{x^2\cdot(x-1-1)\cdot(x-1+1)}{(x-1)^2}=\frac{x^2\cdot(x-2)\cdot x}{(x-1)^2}=$$

$$=\frac{x^3\cdot(x-2)}{(x-1)^2}$$

8.- Realizar:

$$\frac{x}{1+\dfrac{1}{1+\dfrac{1}{x}}}=$$

$$\frac{x}{1+\dfrac{1}{\dfrac{x+1}{x}}}=\frac{x}{1+\dfrac{x}{x+1}}=\frac{x}{\dfrac{x+1+x}{x+1}}=\frac{x}{\dfrac{2x+1}{x+1}}=\frac{x(x+1)}{2x+1}$$