

EJERCICIOS RESUELTOS DE ECUACIONES

1. Resolver:

$$a) \frac{3x+1}{7} - \frac{2-4x}{3} = \frac{-5x-4}{14} + \frac{7x}{6}$$

$$m.c.m.(7, 3, 14, 6) = 42$$

$$6(3x+1) - 14(2-4x) = 3(-5x-4) + 49x$$

$$18x + 6 - 28 + 56x = -15x - 12 + 49x$$

$$18x + 56x + 15x - 49x = -12 - 6 + 28$$

$$40x = 10 \qquad 4x = 1 \qquad x = \frac{1}{4}$$

$$b) \frac{5}{x-7} = \frac{3}{x-2}$$

$$5(x-2) = 3(x-7)$$

$$5x - 10 = 3x - 21$$

$$5x - 3x = -21 + 10$$

$$2x = -11 \qquad x = -\frac{11}{2}$$

$$c) 6\left(\frac{x+1}{8} - \frac{2x-3}{16}\right) = 3\left(\frac{3}{4}x - \frac{1}{4}\right) - \frac{3}{8}(3x-2)$$

$$\frac{6(x+1)}{8} - \frac{6(2x-3)}{16} = \frac{9}{4}x - \frac{3}{4} - \frac{9}{8}x + \frac{6}{8}$$

$$\frac{6x+6}{8} - \frac{12x-18}{16} = \frac{9}{4}x - \frac{3}{4} - \frac{9}{8}x + \frac{6}{8}$$

$$m.c.m.(8, 16, 4) = 16$$

$$2(6x+6) - (12x-18) = 36x - 12 - 18x + 12$$

$$~~12x~~ + 12 - ~~12x~~ + 18 = 36x - ~~12~~ - 18x + ~~12~~$$

$$12 + 18 = 36x - 18x$$

$$18x = 30 \quad 3x = 5 \quad x = \frac{5}{3}$$

$$d) \quad 2 - \left[-2 \cdot (x+1) - \frac{x-3}{2} \right] = \frac{2x}{3} - \frac{5x-3}{12} + 3x$$

Quitamos corchete:

$$2 - \left(-2x - 2 - \frac{x-3}{2} \right) = \frac{2x}{3} - \frac{5x-3}{12} + 3x$$

Quitamos paréntesis:

$$2 + 2x + 2 + \frac{x-3}{2} = \frac{2x}{3} - \frac{5x-3}{12} + 3x$$

Quitamos denominadores:

$$24 + 24x + 24 + 6 \cdot (x-3) = 8x - (5x-3) + 36x$$

Quitamos paréntesis:

$$24 + 24x + 24 + 6x - 18 = 8x - 5x + 3 + 36x$$

Agrupamos términos:

$$24x + 6x - 8x + 5x - 36x = 3 - 24 - 24 + 18$$

Sumamos:

$$-9x = -27$$

Dividimos los dos miembros por: -9

$$x = 3$$

$$e) \quad \frac{2}{3} \left[x - \left(1 - \frac{x-2}{3} \right) \right] + 1 = x$$

$$\frac{2}{3} \left(x - 1 + \frac{x-2}{3} \right) + 1 = x$$

$$\frac{2}{3}x - \frac{2}{3} + \frac{2x-4}{9} + 1 = x$$

$$6x - 6 + 2x - 4 + 9 = 9x$$

$$-x = 1 \quad x = -1$$

2. Resolver:

a) $2x - 3 = 1 - 2x + x^2$

$$x^2 - 4x + 4 = 0$$

$$x = \frac{4 \pm \sqrt{16 - 16}}{2} = \frac{4}{2} = 2$$

b) $x^2 + (7 - x)^2 = 25$

$$x^2 + 49 - 14x + x^2 = 25$$

$$2x^2 - 14x + 24 = 0$$

$$x^2 - 7x + 12 = 0$$

$$x = \frac{7 \pm \sqrt{49 - 48}}{2} = \frac{7 \pm 1}{2} = \begin{cases} \nearrow x_1 = 4 \\ \searrow x_2 = 3 \end{cases}$$

c) $-x^2 + 4x - 7 = 0$

$$x^2 - 4x + 7 = 0$$

$$x = \frac{4 \pm \sqrt{16 - 28}}{2} = \frac{-3 \pm \sqrt{-12}}{2} \notin \mathbb{R}$$

d) $18 = 6x + x(x - 13)$

$$18 = 6x + x^2 - 13x$$

$$x^2 - 7x - 18 = 0$$

$$x = \frac{7 \pm \sqrt{49 + 72}}{2} = \frac{7 \pm \sqrt{121}}{2} = \frac{7 \pm 11}{2} = \begin{cases} \nearrow x_1 = \frac{18}{2} = 9 \\ \searrow x_2 = \frac{-4}{2} = -2 \end{cases}$$

e) $x^2 - \frac{7}{6}x + \frac{1}{3} = 0$

$$6x^2 - 7x + 2 = 0$$

$$x = \frac{7 \pm \sqrt{7^2 - 4 \cdot 6 \cdot 2}}{12} = \frac{7 \pm \sqrt{49 - 48}}{12} = \frac{7 \pm \sqrt{1}}{12} = \frac{7 \pm 1}{12} = \begin{cases} \nearrow x_1 = \frac{8}{12} = \frac{2}{3} \\ \searrow x_2 = \frac{6}{12} = \frac{1}{2} \end{cases}$$

$$f) 2x^2 - 6x = 0$$

$$2x(x - 3) = 0$$

$$2x = 0 \quad x = 0$$

$$x - 3 = 0 \quad x = 3$$

$$g) 2x^2 + 8 = 0$$

$$2x^2 = -8 \quad x^2 = -4 \quad x = \pm\sqrt{-4} \notin \mathbb{R}$$

$$h) 4x^2 - 16 = 0$$

$$4x^2 = 16 \quad x^2 = 4 \quad x = \pm\sqrt{4} \begin{cases} \nearrow x_1 = 2 \\ \searrow x_2 = -2 \end{cases}$$

$$i) 6x^2 + 3x = 0$$

$$3x(2x + 1) = 0 \quad 3x = 0 \quad x = 0$$

$$2x + 1 = 0 \quad x = -\frac{1}{2}$$

3. Resolver:

$$a) x^4 + 12x^3 - 64x^2 = 0$$

$$x^2(x^2 + 12x - 64) = 0$$

$$x^2 = 0 \quad x_1 = 0$$

$$x^2 + 12x - 64 = 0$$

$$x = \frac{-12 \pm \sqrt{144 + 256}}{2} = \frac{-12 \pm 20}{2} = \begin{cases} \nearrow x_2 = 4 \\ \searrow x_3 = -16 \end{cases}$$

$$b) 2x^3 - 7x^2 + 8x - 3 = 0$$

$$P(1) = 2 \cdot 1^3 - 7 \cdot 1^2 + 8 \cdot 1 - 3 = 0$$

	2	-7	8	-3
1		2	-5	3
	2	-5	3	0

$$\begin{array}{cccc} 1 & 3 & -4 & -12 \\ 2 & & 2 & 10 & 12 \\ \hline 1 & 5 & 6 & 0 \end{array}$$

$$(x - 2) \cdot (x^2 - 5x + 6) = 0$$

$$x^2 - 5x + 6 = 0$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 6}}{2} = \frac{-5 \pm \sqrt{25 - 24}}{2} = \frac{-5 \pm \sqrt{1}}{2} = \frac{-5 \pm 1}{2} = \begin{array}{l} \nearrow x_1 = \frac{-4}{2} = -2 \\ \searrow x_2 = \frac{-6}{2} = -3 \end{array}$$

$$(x - 2) \cdot (x + 2) \cdot (x + 3) = 0$$

Las soluciones son : $x = 2, x = -2, x = -3$.

$$e) x^4 - 16x^2 - 225 = 0$$

$$x^2 = t$$

$$t^2 - 16t - 225 = 0$$

$$t = \frac{16 \pm \sqrt{256 + 900}}{2} = \frac{16 \pm 34}{2} = \begin{array}{l} \nearrow t_1 = 25 \\ \searrow t_2 = -9 \end{array}$$

$$x^2 = 25 \quad x = \pm\sqrt{25} \begin{array}{l} \nearrow x_1 = 5 \\ \searrow x_2 = -5 \end{array}$$

$$x^2 = -9 \quad x = \pm\sqrt{-9} \notin \mathbb{R}$$

$$f) x^6 - 7x^3 + 6 = 0$$

$$x^3 = t$$

$$t^2 - 7t + 6 = 0$$

$$t = \frac{7 \pm \sqrt{49 - 24}}{2} = \frac{7 \pm 5}{2} = \begin{array}{l} \nearrow t_1 = \frac{12}{2} = 6 \\ \searrow t_2 = \frac{2}{2} = 1 \end{array}$$

$$x^3 = 6 \quad x = \sqrt[3]{6}$$

$$x^3 = 1 \quad x = \sqrt[3]{1} \quad x = 1$$

$$g) x^4 - 61x^2 + 900 = 0$$

$$x^2 = t$$

$$t^2 - 61t + 900 = 0$$

$$t = \frac{61 \pm \sqrt{3721 - 3600}}{2} = \frac{61 \pm 11}{2} = \begin{cases} \nearrow t_1 = 36 \\ \searrow t_2 = 25 \end{cases}$$

$$x^2 = 36 \quad x = \pm\sqrt{36} \begin{cases} \nearrow x_1 = 6 \\ \searrow x_2 = -6 \end{cases}$$

$$x^2 = 25 \quad x = \pm\sqrt{25} \begin{cases} \nearrow x_1 = 5 \\ \searrow x_2 = -5 \end{cases}$$

4. Resolver las siguientes ecuaciones racionales:

$$a) \frac{1}{x^2 - x} - \frac{1}{x - 1} = 0$$

$$x^2 - x = x(x - 1)$$

$$m.c.m.(x^2 - x, x - 1) = x(x - 1)$$

$$1 - x = 0 \quad x = 1$$

Comprobamos la solución:

$$\frac{1}{1-1} - \frac{1}{1-1} = 0 \quad \frac{1}{0} - \frac{1}{0} = 0$$

La ecuación no tiene solución porque para $x = 1$ se anulan los denominadores.

$$b) \frac{1}{x-2} + \frac{1}{x+2} = \frac{1}{x^2-4}$$

$$x^2 - 4 = (x - 2) \cdot (x + 2)$$

$$m.c.m.(x - 2, x + 2, x^2 - 4) = (x - 2) \cdot (x + 2)$$

$$x + 2 + x - 2 = 1 \quad 2x = 1 \quad x = \frac{1}{2}$$

$$\frac{1}{\frac{1}{2}-2} + \frac{1}{\frac{1}{2}+2} = \frac{1}{\left(\frac{1}{2}\right)^2 - 4}$$

$$\frac{\frac{1}{-3}}{\frac{-2}{2}} + \frac{\frac{1}{5}}{\frac{2}{2}} = \frac{1}{\frac{-15}{4}} \quad -\frac{2}{3} + \frac{2}{5} = -\frac{4}{15} \quad -\frac{4}{15} = -\frac{4}{15}$$

La solución es: $x = \frac{1}{2}$

c) $\frac{3}{x} = 1 + \frac{x-13}{6}$

$$m.c.m.(x, 6) = 6x$$

$$18 = 6x + x(x-13)$$

$$18 = 6x + x^2 - 13x$$

$$x^2 - 7x - 18 = 0$$

$$x = \frac{7 \pm \sqrt{49+72}}{2} = \frac{7 \pm \sqrt{121}}{2} = \frac{7 \pm 11}{2} = \begin{matrix} \nearrow x_1 = \frac{18}{2} = 9 \\ \searrow x_2 = \frac{-4}{2} = -2 \end{matrix}$$

$$\frac{3}{9} = 1 + \frac{9-13}{6} \quad \frac{3}{9} = \frac{6-4}{6} \quad \frac{3}{9} = \frac{2}{6} \quad \frac{1}{3} = \frac{1}{3}$$

$$\frac{3}{-2} = 1 + \frac{-2-13}{6} \quad \frac{3}{-2} = \frac{6-15}{6} \quad \frac{3}{-2} = \frac{-9}{6} \quad -\frac{3}{2} = -\frac{3}{2}$$

5. Resolver las siguientes ecuaciones irracionales:

a) $\sqrt{2x-3} = -1+x$

Elevamos al cuadrado los dos miembros:

$$(\sqrt{2x-3})^2 = (-1+x)^2$$

$$2x-3 = 1-2x+x^2$$

Resolvemos la ecuación:

$$x^2 - 4x + 4 = 0$$

$$x = \frac{4 \pm \sqrt{16-16}}{2} = \frac{4}{2} = 2$$

Comprobamos:

$$\sqrt{2 \cdot 2 - 3} - 2 = -1 \quad 1 - 2 = -1$$

La ecuación tiene por solución $x = 2$.

b) $\sqrt{5x+4} - 1 = 2x$

$$\sqrt{5x+4} = 2x+1 \quad (\sqrt{5x+4})^2 = (2x+1)^2$$

$$5x+4 = 4x^2+4x+1 \quad 4x^2-x-3=0$$

$$x = \frac{1 \pm \sqrt{1+48}}{8} = \frac{1 \pm 7}{8} = \begin{matrix} \nearrow x_1 = \frac{8}{8} = 1 \\ \searrow x_2 = \frac{-6}{8} = -\frac{3}{4} \end{matrix}$$

$$\sqrt{5 \cdot 1 + 4} - 1 = 2 \cdot 1 \quad 3 - 1 = 2 \quad x = 1$$

$$\sqrt{5 \cdot \left(-\frac{3}{4}\right) + 4} - 1 = 2 \cdot \left(-\frac{3}{4}\right) \quad -\frac{1}{2} \neq -\frac{3}{2}$$

c) $3\sqrt{x-1} + 11 = 2x$

$$3\sqrt{x-1} = 2x-11 \quad (3\sqrt{x-1})^2 = (2x-11)^2$$

$$9(x-1) = 4x^2 - 44x + 121 \quad 9x - 9 = 4x^2 - 44x + 121$$

$$4x^2 - 53x + 130 = 0$$

$$x = \frac{53 \pm \sqrt{2809 - 2080}}{8} = \frac{53 \pm \sqrt{729}}{8} = \frac{53 \pm 27}{8} = \begin{matrix} \nearrow x_1 = \frac{80}{8} = 10 \\ \searrow x_2 = \frac{26}{8} = \frac{13}{4} \end{matrix}$$

$$3\sqrt{10-1} + 11 = 2 \cdot 10 \quad 20 = 20$$

$$3\sqrt{\frac{13}{4}-1} + 11 \neq 2 \cdot \frac{13}{4}$$

d) $\sqrt{x} + \sqrt{x-4} = 2$

$$\sqrt{x} = 2 - \sqrt{x-4}$$

$$(\sqrt{x})^2 = (2 - \sqrt{x-4})^2$$

$$x = 4 - 4\sqrt{x-4} + x - 4$$

$$4\sqrt{x-4} - 0 \quad \sqrt{x-4} - 0$$

$$(\sqrt{x-4})^2 = 0^2 \quad x - 4 = 0 \quad x = 4$$

$$\sqrt{4} + \sqrt{4-4} = 2 \quad 2 + 0 = 2$$

La ecuación tiene por solución $x = 4$.

$$e) \sqrt{2x-1} + \sqrt{x+4} = 6$$

$$\sqrt{2x-1} = 6 - \sqrt{x+4}$$

$$(\sqrt{2x-1})^2 = (6 - \sqrt{x+4})^2$$

$$2x - 1 = 36 - 12\sqrt{x+4} + x + 4$$

$$x - 41 = -12\sqrt{x+4}$$

$$(x - 41)^2 = (-12\sqrt{x+4})^2$$

$$x^2 - 82x + 1681 = 144x + 576$$

$$x^2 - 226x + 1105 = 0 \quad \begin{array}{l} \nearrow x_1 = 5 \\ \searrow x_2 = 221 \end{array}$$

$$\sqrt{2 \cdot 5 - 1} + \sqrt{5 + 4} = 6 \quad 3 + 3 = 6 \quad x = 5$$

$$\sqrt{2 \cdot 221 - 1} + \sqrt{221 + 4} = 6 \quad 21 + 15 \neq 6$$

6. Resolver los siguientes sistemas lineales:

$$a) \begin{cases} \frac{x+y}{2} = x-1 \\ \frac{x-y}{2} = y+1 \end{cases}$$

$$\begin{cases} x + y = 2(x - 1) \\ x - y = 2(y + 1) \end{cases}$$

$$\begin{cases} x + y = 2x - 2 \\ x - y = 2y + 2 \end{cases}$$

$$\begin{cases} -x + y = -2 \\ x - 3y = 2 \end{cases}$$

$$\begin{array}{r} -x + y = -2 \\ x - 3y = 2 \\ \hline -2y = 0 \end{array} \quad y = 0$$

$$x - 3 \cdot 0 = 2 \quad x = 2$$

$$b) \begin{cases} \frac{x + 3y}{2} = 5 \\ 3x - y = 5y \end{cases}$$

$$\begin{cases} x + 3y = 10 \\ 3x - y = 5y \end{cases}$$

$$\begin{array}{r} x = 10 - 3y \\ 3x = 6y \end{array} \quad x = 2y$$

$$10 - 3y = 2y \quad y = 2$$

$$x = 2 \cdot 2 \quad x = 4$$

$$c) \begin{cases} \frac{x + 3y}{2} = 5 \\ 4 - \frac{2x - y}{2} = 1 \end{cases}$$

$$\begin{cases} x + 3y = 10 \\ 8 - 2x + y = 2 \end{cases}$$

$$\begin{cases} x + 3y = 10 \\ -2x + y = -6 \end{cases}$$

$$x = 10 - 3y$$

$$-2(10 - 3y) + y = -6 \quad -20 + 6y + y = -6 \quad 7y = 14 \quad y = 2$$

$$x = 10 - 3 \cdot 2 \quad x = 4$$

$$d) \begin{cases} \frac{x}{2} + \frac{y}{3} = 4 \\ \frac{x}{3} + y = 1 \end{cases}$$

$$\begin{cases} 3x + 2y = 24 \\ x + 3y = 3 \end{cases}$$

$$\begin{array}{r} 3x + 2y = 24 \\ -3x - 9y = -9 \\ \hline -7y = 15 \end{array} \quad y = -\frac{15}{7}$$

$$x + 3\left(-\frac{15}{7}\right) = 3 \quad x - \frac{45}{7} = 3$$

$$7x - 45 = 21 \quad 7x = 66 \quad x = \frac{66}{7}$$

7. Resolver los siguientes sistemas no lineales:

$$a) \begin{cases} x^2 + y^2 = 25 \\ x + y = 7 \end{cases}$$

$$y = 7 - x$$

$$x^2 + (7 - x)^2 = 25$$

$$x^2 + 49 - 14x + x^2 = 25$$

$$2x^2 - 14x + 24 = 0$$

$$x^2 - 7x + 12 = 0$$

$$x = \frac{7 \pm \sqrt{49 - 48}}{2} = \frac{7 \pm 1}{2} = \begin{cases} \nearrow x_1 = 4 \\ \searrow x_2 = 3 \end{cases}$$

$$x = 3 \quad y = 7 - 3 \quad y = 4$$

$$x = 4 \quad y = 7 - 4 \quad y = 3$$

$$b) \begin{cases} x + y = 7 \\ x \cdot y = 12 \end{cases}$$

$$y = 7 - x$$

$$x \cdot (7 - x) = 12 \quad 7x - x^2 = 12 \quad x^2 - 7x + 12 = 0$$

$$x = \frac{7 \pm \sqrt{49 - 48}}{2} = \frac{7 \pm 1}{2} \begin{matrix} \nearrow x_1 = 4 \\ \searrow x_2 = 3 \end{matrix}$$

$$x_1 = 4 \quad y = 7 - 4 \quad y_1 = 3$$

$$x_2 = 3 \quad y = 7 - 3 \quad y_2 = 4$$

$$c) \begin{cases} x^2 + y^2 = 169 \\ x + y = 17 \end{cases}$$

$$x = 17 - y$$

$$2y^2 - 34y + 120 = 0 \quad y^2 - 17y + 60 = 0$$

$$y = \frac{17 \pm \sqrt{289 - 240}}{2} = \frac{17 \pm 7}{2} \begin{matrix} \nearrow y_1 = 12 \\ \searrow y_2 = 5 \end{matrix}$$

$$y_1 = 12 \quad x_1 = 17 - 12 \quad x_1 = 5$$

$$y_2 = 5 \quad x_1 = 17 - 5 \quad x_2 = 12$$

$$d) \begin{cases} y^2 - 2y + 1 = x \\ \sqrt{x} + y = 5 \end{cases}$$

$$\sqrt{y^2 - 2y + 1} + y = 5 \quad (\sqrt{y^2 - 2y + 1})^2 = (5 - y)^2$$

$$y^2 - 2y + 1 = 25 - 10y + y^2 \quad 8y = 24 \quad y = 3$$

$$x = 3^2 - 2 \cdot 3 + 1 \quad x = 4$$