EJERCICIOS DE INTEGRALES INDEFINIDAS

Ejercicio 1:

Calcula las siguientes integrales:

a) $\int 7x^4$ b) $\int \frac{1}{x^2}$ c) $\int \sqrt{x}$

d) $\int \sqrt[3]{5x^2}$ e) $\int \frac{\sqrt[3]{x} + \sqrt{5x^3}}{3x}$ f) $\int \frac{\sqrt{5x^3}}{\sqrt[3]{3x}}$

a) $\int 7x^4 = 7\,\frac{x^5}{5} + k = \frac{7x^5}{5} + k$

b) $\int \frac{1}{x^2} = \int x^{-2} = \frac{x^{-1}}{-1} + k = \frac{-1}{x} + k$

c) $\int \sqrt{x} = \int x^{1/2} = \frac{x^{3/2}}{3/2} + k = \frac{2\sqrt{x^3}}{3} + k$

d) $\int \sqrt[3]{5x^2} = \int \sqrt[3]{5}\,x^{2/3} = \sqrt[3]{5}\,\frac{x^{5/3}}{5/3} + k = \frac{3\,\sqrt[3]{5x^5}}{5} + k$

e) $\int \frac{\sqrt[3]{x} + \sqrt{5x^3}}{3x} = \int \frac{x^{1/3}}{3x} + \int \frac{\sqrt{5}\,x^{3/2}}{3x} = \frac{1}{3}\int x^{-2/3} + \frac{\sqrt{5}}{3}\int x^{1/2} =$

$= \frac{1}{3}\,\frac{x^{1/3}}{1/3} + \frac{\sqrt{5}}{3}\,\frac{x^{3/2}}{3/2} + k = \sqrt[3]{x} + \frac{2\sqrt{5x^3}}{9} + k$

f) $\int \frac{\sqrt{5x^3}}{\sqrt[3]{3x}} = \int \frac{\sqrt{5}\cdot x^{3/2}}{\sqrt[3]{3}\cdot x^{1/3}} = \frac{\sqrt{5}}{\sqrt[3]{3}}\int x^{7/6} = \frac{\sqrt{5}}{\sqrt[3]{3}}\,\frac{x^{13/6}}{13/6} + k = \frac{6\sqrt{5}\,\sqrt[6]{x^{13}}}{13\sqrt[3]{3}} + k$

Ejercicio 2:

Calcula:

a) $\int \frac{x^4 - 5x^2 + 3x - 4}{x}$ b) $\int \frac{x^4 - 5x^2 + 3x - 4}{x + 1}$

c) $\int \frac{x^4 - 5x^2 + 3x - 4}{x^2 + 1}$ d) $\int \frac{x^3}{x - 2}$

a) $\int \frac{x^4 - 5x^2 + 3x - 4}{x} = \int \left(x^3 - 5x + 3 - \frac{4}{x}\right) = \frac{x^4}{4} - \frac{5x^2}{2} + 3x - 4\ ln\ |x| + k$

b) $\int \frac{x^4 - 5x^2 + 3x - 4}{x + 1} = \int \left(x^3 - x^2 - 4x + 7 - \frac{11}{x + 1}\right) =$

$$= \frac{x^4}{4} - \frac{x^3}{3} - 2x^2 + 7x - 11\ ln\ |x + 1| + k$$

c) $\int \frac{x^4 - 5x^2 + 3x - 4}{x^2 + 1} = \int \left(x^2 - 6 + \frac{3x + 2}{x^2 + 1}\right) = \int \left(x^2 - 6 + \frac{3x}{x^2 + 1} + \frac{2}{x^2 + 1}\right) =$

$$= \int x^2 - \int 6 + \frac{3}{2} \int \frac{2x}{x^2 + 1} + 2 \int \frac{1}{x^2 + 1} =$$

$$= \frac{x^3}{3} - 6x + \frac{3}{2}\ ln\ (x^2 + 1) + 2\ arc\ tg\ x + k$$

d) $\int \frac{x^3}{x - 2} = \int \left(x^2 + 2x + 4 + \frac{8}{x - 2}\right) = \frac{x^3}{3} + x^2 + 4x + 8\ ln\ |x - 2| + k$

Ejercicio 3:

Calcula:

a) $\int cos^4\ x\ sen\ x\ dx$ **b)** $\int 2^{sen\ x} cos\ x\ dx$

a) $\int cos^4\ x\ sen\ x\ dx = -\int cos^4\ x (-sen\ x)\ dx = -\frac{cos^5\ x}{5} + k$

b) $\int 2^{sen\ x} cos\ x\ dx = \frac{1}{ln\ 2} \int 2^{sen\ x} cos\ x \cdot ln\ 2\ dx = \frac{2^{sen\ x}}{ln\ 2} + k$

Ejercicio 4:

Calcula:

a) $\int cotg\ x\ dx$ **b)** $\int \frac{5x}{x^4 + 1} dx$

a) $\int cotg\ x\ dx = \int \frac{cos\ x}{sen\ x} dx = ln\ |sen\ x| + k$

b) $\int \frac{5x}{x^4 + 1} dx = \frac{5}{2} \int \frac{2x}{1 + (x^2)^2} dx = \frac{5}{2}\ arc\ tg\ (x^2) + k$

Ejercicio 5:

Calcula: $\int \frac{1}{\sqrt[3]{x^2} - \sqrt{x}}\, dx$

Hacemos el cambio $x = t^6$, $dx = 6t^5\, dt$:

$$\int \frac{1}{\sqrt[3]{x^2} - \sqrt{x}}\, dx = \int \frac{1}{\sqrt[3]{t^{12}} - \sqrt{t^6}}\, 6t^5\, dt = \int \frac{6t^5}{t^4 - t^3}\, dt = \int \frac{6t^2}{t-1}\, dt = 6\int \frac{t^2}{t-1}\, dt =$$

$$= 6\int \left(t + 1 + \frac{1}{t-1}\right) dt = 6\int \left(\frac{t^2}{2} + t - ln\, |t-1|\right) + k =$$

$$= 6\left(\frac{\sqrt[6]{x^2}}{2} + \sqrt[6]{x} - ln\, |\sqrt[6]{x} - 1|\right) + k = 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6\, ln\, |\sqrt[6]{x} - 1| + k$$

Ejercicio 6:

Calcula: $\int \frac{x}{\sqrt{1 - x^2}}\, dx$

Hacemos el cambio $\sqrt{1 - x^2} = t \;\rightarrow\; 1 - x^2 = t^2 \;\rightarrow\; x = \sqrt{1 - t^2}$

$$dx = \frac{-t}{\sqrt{1 - t^2}}\, dt$$

$$\int \frac{x}{\sqrt{1 - x^2}}\, dx = \int \frac{\sqrt{1 - t^2}}{t^2} \cdot \frac{-t}{\sqrt{1 - t^2}}\, dt = \int -1\, dt = -t + k = -\sqrt{1 - x^2} + k$$

Ejercicio 7:

Calcula: $\int x\, sen\, x\, dx$

Llamamos $I = \int x\, sen\, x\, dx$.

$$\left.\begin{array}{l} u = x, \quad du = dx \\ dv = sen\, x\, dx, \quad v = -cos\, x \end{array}\right\} \; I = -x\, cos\, x + \int cos\, x\, dx = -x\, cos\, x + sen\, x + k$$

Ejercicio 8:

Calcula: $\int x\, arc\, tg\, x\, dx$

Llamamos $I = \int x\, arc\, tg\, x\, dx$.

$$\left.\begin{array}{l} u = arc\, tg\, x, \quad du = \dfrac{1}{1 + x^2}\, dx \\ dv = x\, dx, \quad v = \dfrac{x^2}{2} \end{array}\right\}$$

$$I = \frac{x^2}{2}\, arc\, tg\, x - \frac{1}{2}\int \left(\frac{x^2}{1 + x^2}\right) dx = \frac{x^2}{2}\, arc\, tg\, x - \frac{1}{2}\int \left(1 - \frac{1}{1 + x^2}\right) dx =$$

$$= \frac{x^2}{2}\, arc\, tg\, x - \frac{1}{2}[x - arc\, tg\, x] + k = \frac{x^2}{2}\, arc\, tg\, x - \frac{1}{2}x + \frac{1}{2}\, arc\, tg\, x + k =$$

$$= \frac{x^2 + 1}{2}\, arc\, tg\, x - \frac{1}{2}x + k$$

Ejercicio 9:

Calcula: $\int \frac{3x^2-5x+1}{x-4}\,dx$

$$\int \frac{3x^2-5x+1}{x-4}\,dx = \int\left(3x+7+\frac{29}{x-4}\right)dx = \frac{3x^2}{2}+7x+29\,ln\,|x-4|+k$$

Ejercicio 10:

Calcula:

a) $\int \frac{5x-3}{x^3-x}\,dx$ b) $\int \frac{x^2-2x+6}{(x-1)^3}\,dx$

a) Descomponemos la fracción:

$$\frac{5x-3}{x^3-x} = \frac{5x-3}{x(x-1)(x+1)} = \frac{A}{x}+\frac{B}{x-1}+\frac{C}{x+1}$$

$$\frac{5x-3}{x^3-x} = \frac{A(x-1)(x+1)+Bx(x+1)+Cx(x-1)}{x(x-1)(x+1)}$$

$$5x-3 = A(x-1)(x+1)+Bx(x+1)+Cx(x-1)$$

Hallamos A, B y C dando a x los valores 0, 1 y –1:

$$\left.\begin{array}{lllll} x=0 & \Rightarrow & -3=-A & \Rightarrow & A=3 \\ x=1 & \Rightarrow & 2=2B & \Rightarrow & B=1 \\ x=-1 & \Rightarrow & -8=2C & \Rightarrow & C=-4 \end{array}\right\}$$

Así, tenemos que:

$$\int \frac{5x-3}{x^3-x}\,dx = \int\left(\frac{3}{x}+\frac{1}{x-1}-\frac{4}{x+1}\right)dx = 3\,ln|x|+ln|x-1|-4\,ln|x+1|+k$$

b) Descomponemos la fracción:

$$\frac{x^2-2x+6}{(x-1)^3} = \frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{(x-1)^3} = \frac{A(x-1)^2+B(x-1)+C}{(x-1)^3}$$

$$x^2-2x+6 = A(x-1)^2+B(x-1)+C$$

Dando a x los valores 1, 0 y 2, queda:

$$\left.\begin{array}{lll} x=1 & \Rightarrow & 5=C \\ x=0 & \Rightarrow & 6=A-B+C \\ x=2 & \Rightarrow & 6=A+B+C \end{array}\right\}\begin{array}{l} A=1 \\ B=0 \\ C=5 \end{array}$$

Por tanto:

$$\int \frac{x^2-2x+6}{(x-1)^3}\,dx = \int\left(\frac{1}{x-1}+\frac{5}{(x-1)^3}\right)dx = ln|x-1|-\frac{5}{2(x-1)^2}+k$$

Ejercicio 11:

Calcula:

a) $\int \frac{x^3 + 22x^2 - 12x + 8}{x^4 - 4x^2}\, dx$ **b)** $\int \frac{x^3 - 4x^2 + 4x}{x^4 - 2x^3 - 4x^2 + 8x}\, dx$

a) $x^4 - 4x^2 = x^2(x^2 - 4) = x^2(x - 2)(x + 2)$

Descomponemos la fracción:

$$\frac{x^3 + 22x^2 - 12x + 8}{x^2(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2} + \frac{D}{x+2}$$

$$\frac{x^3 + 22x^2 - 12x + 8}{x^2(x-2)(x+2)} =$$

$$= \frac{Ax(x-2)(x+2) + B(x-2)(x+2) + Cx^2(x+2) + Dx^2(x-2)}{x^2(x-2)(x+2)}$$

$$x^3 + 22x^2 - 12x + 8 = Ax(x-2)(x+2) + B(x-2)(x+2) + Cx^2(x+2) + Dx^2(x-2)$$

Hallamos A, B, C y D dando a x los valores 0, 2, –2 y 1:

$$\left.\begin{array}{llllllll} x = 0 & \Rightarrow & 8 = -4B & \Rightarrow & B = -2 & & & \\ x = 2 & \Rightarrow & 80 = 16C & \Rightarrow & C = 5 & & & \\ x = -2 & \Rightarrow & 112 = -16D & \Rightarrow & D = -7 & & & \\ x = 1 & \Rightarrow & 19 = -3A - 3B + 3C - D & \Rightarrow & -3A = -9 & \Rightarrow & A = 3 & \end{array}\right\}$$

Por tanto:

$$\int \frac{x^3 + 22x^2 - 12x + 8}{x^4 - 4x^2}\, dx = \int \left(\frac{3}{x} - \frac{2}{x^2} + \frac{5}{x-2} - \frac{7}{x+2} \right) dx =$$

$$= 3\, ln|x| + \frac{2}{x} + 5\, ln|x-2| - 7\, ln|x+2| + k$$

b) La fracción se puede simplificar:

$$\frac{x^3 - 4x^2 + 4x}{x^4 - 2x^3 - 4x^2 + 8x} = \frac{x(x-2)^2}{x(x-2)^2(x+2)} = \frac{1}{x+2}$$

$$\int \frac{x^3 - 4x^2 + 4x}{x^4 - 2x^3 - 4x^2 + 8x}\, dx = \int \frac{1}{x+2}\, dx = ln|x+2| + k$$

Ejercicio 12:

Calcula las siguientes integrales inmediatas:

a) $\int (4x^2 - 5x + 7)\, dx$ **b)** $\int \frac{dx}{\sqrt[5]{x}}$ **c)** $\int \frac{1}{2x+7}\, dx$ **d)** $\int (x - sen\, x)\, dx$

a) $\int (4x^2 - 5x + 7)\, dx = \frac{4x^3}{3} - \frac{5x^2}{2} + 7x + k$

b) $\int \frac{dx}{\sqrt[5]{x}} = \int x^{-1/5}\, dx = \frac{x^{4/5}}{4/5} + k = \frac{5\sqrt[5]{x^4}}{4} + k$

c) $\int \frac{1}{2x+7}\,dx = \frac{1}{2}\,ln|2x+7| + k$

d) $\int (x - sen\ x)\,dx = \frac{x^2}{2} + cox\ x + k$

Ejercicio 13:

Resuelve estas integrales:

a) $\int (x^2 + 4x)(x^2 - 1)\,dx$ **b)** $\int (x-1)^3\,dx$

c) $\int \sqrt{3x}\,dx$ **d)** $\int (sen\ x + e^x)\,dx$

a) $\int (x^2 + 4x)(x^2 - 1)\,dx = \int (x^4 + 4x^3 - x^2 - 4x)\,dx = \frac{x^5}{5} + x^4 - \frac{x^3}{3} - 2x^2 + k$

b) $\int (x-1)^3\,dx = \frac{(x-1)^4}{4} + k$

c) $\int \sqrt{3x}\,dx = \int \sqrt{3}\,x^{1/2}\,dx = \sqrt{3}\,\frac{x^{3/2}}{3/2} + k = \frac{2\sqrt{3x^3}}{3} + k$

d) $\int (sen\ x + e^x)\,dx = -cos\ x + e^x + k$

Ejercicio 14:

Calcula las integrales siguientes:

a) $\int \sqrt[3]{\frac{x}{2}}\,dx$ **b)** $\int sen(x-4)\,dx$ **c)** $\int \frac{7}{cos^2 x}\,dx$ **d)** $\int (e^x + 3e^{-x})\,dx$

a) $\int \sqrt[3]{\frac{x}{2}}\,dx = \frac{1}{\sqrt[3]{2}} \int x^{1/3}\,dx = \frac{1}{\sqrt[3]{2}}\,\frac{x^{4/3}}{4/3} + k = \frac{3}{4}\sqrt[3]{\frac{x^4}{2}} + k$

b) $\int sen\,(x-4)\,dx = -cos\,(x-4) + k$

c) $\int \frac{7}{cos^2 x}\,dx = 7\ tg\ x + k$

d) $\int (e^x + 3e^{-x})\,dx = e^x - 3e^{-x} + k$

Ejercicio 15:

Halla estas integrales:

a) $\int \frac{2}{x}\,dx$ **b)** $\int \frac{dx}{x-1}$ **c)** $\int \frac{x + \sqrt{x}}{x^2}\,dx$ **d)** $\int \frac{3}{1+x^2}\,dx$

a) $\int \frac{2}{x}\,dx = 2\,ln|x| + k$

b) $\int \frac{dx}{x-1} = ln|x-1| + k$

c) $\int \frac{x+\sqrt{x}}{x^2}\,dx = \int \left(\frac{1}{x} + x^{-3/2}\right) dx = ln|x| - \frac{2}{\sqrt{x}} + k$

d) $\int \frac{3}{1+x^2}\,dx = 3\ arc\ tg\ x + k$

Ejercicio 16:

Resuelve las siguientes integrales:

a) $\int \frac{dx}{x-4}$ **b)** $\int \frac{dx}{(x-4)^2}$ **c)** $\int (x-4)^2 dx$ **d)** $\int \frac{dx}{(x-4)^3}$

a) $\int \frac{dx}{x-4} = ln|x-4| + k$

b) $\int \frac{dx}{(x-4)^2} = \frac{-1}{(x-4)} + k$

c) $\int (x-4)^2 dx = \frac{(x-4)^3}{3} + k$

d) $\int \frac{dx}{(x-4)^3} = \int (x-4)^{-3}\,dx = \frac{(x-4)^{-2}}{-2} + k = \frac{-1}{2(x-4)^2} + k$

Ejercicio 17:

Halla las siguientes integrales del tipo exponencial:

a) $\int e^{x-4}\,dx$ **b)** $\int e^{-2x+9}\,dx$ **c)** $\int e^{5x}\,dx$ **d)** $\int (3^x - x^3)\,dx$

a) $\int e^{x-4}\,dx = e^{x-4} + k$

b) $\int e^{-2x+9}\,dx = \frac{-1}{2}\int -2e^{-2x+9}\,dx = \frac{-1}{2}e^{-2x+9} + k$

c) $\int e^{5x}\,dx = \frac{1}{5}\int 5e^{5x}\,dx = \frac{1}{5}e^{5x} + k$

d) $\int (3^x - x^3)\,dx = \frac{3^x}{ln\ 3} - \frac{x^4}{4} + k$

Ejercicio 18:

Resuelve las siguientes integrales del tipo arco tangente:

a) $\int \frac{dx}{4+x^2}$ **b)** $\int \frac{4\,dx}{3+x^2}$ **c)** $\int \frac{5\,dx}{4x^2+1}$ **d)** $\int \frac{2\,dx}{1+9x^2}$

a) $\int \frac{dx}{4+x^2} = \int \frac{1/4}{1+(x/2)^2}\,dx = \frac{1}{2}\int \frac{1/2}{1+(x/2)^2}\,dx = \frac{1}{2}\ arc\ tg\left(\frac{x}{2}\right) + k$

b) $\int \frac{4\,dx}{3+x^2} = \int \frac{4/3}{1+(x/\sqrt{3})^2}\,dx = \frac{4\sqrt{3}}{3}\int \frac{1/\sqrt{3}}{1+(x/\sqrt{3})^2}\,dx = \frac{4\sqrt{3}}{3}\,arc\ tg\left(\frac{x}{\sqrt{3}}\right) + k$

c) $\int \frac{5\,dx}{4x^2+1} = \frac{5}{2}\int \frac{2\,dx}{(2x)^2+1} = \frac{5}{2}\,arc\ tg\,(2x) + k$

d) $\int \frac{2\,dx}{1+9x^2} = \frac{2}{3}\int \frac{3\,dx}{1+(3x)^2} = \frac{2}{3}\,arc\ tg\,(3x) + k$

Ejercicio 19:

Expresa las siguientes integrales de la forma:

$$\frac{\text{dividendo}}{\text{divisor}} = \text{cociente} + \frac{\text{resto}}{\text{divisor}}$$

y resuélvelas:

a) $\int \frac{x^2-5x+4}{x+1}\,dx$ **b)** $\int \frac{x^2+2x+4}{x+1}\,dx$ **c)** $\int \frac{x^3-3x^2+x-1}{x-2}\,dx$

a) $\int \frac{x^2-5x+4}{x+1}\,dx = \int \left(x - 6 + \frac{10}{x+1}\right)dx = \frac{x^2}{2} - 6x + 10\,ln|x+1| + k$

b) $\int \frac{x^2+2x+4}{x+1}\,dx = \int \left(x + 1 + \frac{3}{x+1}\right)dx = \frac{x^2}{2} + x + 3\,ln|x+1| + k$

c) $\int \frac{x^3-3x^2+x-1}{x-2}\,dx = \int \left(x^2 - x - 1 - \frac{3}{x-2}\right)dx =$

$= \frac{x^3}{3} - \frac{x^2}{2} - x - 3\,ln|x-2| + k$

Ejercicio 20:

Halla estas integrales sabiendo que son del tipo arco seno:

a) $\int \frac{dx}{\sqrt{1-4x^2}}$ **b)** $\int \frac{dx}{\sqrt{4-x^2}}$ **c)** $\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$ **d)** $\int \frac{dx}{x\sqrt{1-(ln\ x)^2}}$

a) $\int \frac{dx}{\sqrt{1-4x^2}} = \frac{1}{2}\int \frac{2\,dx}{\sqrt{1-(2x)^2}} = \frac{1}{2}\,arc\ sen\,(2x) + k$

b) $\int \frac{dx}{\sqrt{4-x^2}} = \int \frac{1/2\,dx}{\sqrt{1-(x/2)^2}} = arc\ sen\left(\frac{x}{2}\right) + k$

c) $\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx = \int \frac{e^x}{\sqrt{1-(e^x)^2}}\,dx = arc\ sen\,(e^x) + k$

d) $\int \frac{dx}{x\sqrt{1-(ln\ x)^2}} = \int \frac{1/x\,dx}{\sqrt{1-(ln\ x)^2}} = arc\ sen\,(ln|x|) + k$

Ejercicio 21:

Resuelve las integrales siguientes,

a) $\int \cos x \, sen^3 x \, dx$ **b)** $\int 2x \, e^{x^2} \, dx$ **c)** $\int \frac{x \, dx}{(x^2+3)^5}$ **d)** $\int \frac{1}{x} \ln^3 x \, dx$

a) $\int \cos x \, sen^3 x \, dx = \frac{sen^4 x}{4} + k$

b) $\int 2x \, e^{x^2} \, dx = e^{x^2} + k$

c) $\int \frac{x \, dx}{(x^2+3)^5} = \frac{1}{2} \int 2x(x^2+3)^{-5} \, dx = \frac{1}{2} \frac{(x^2+3)^{-4}}{-4} + k = \frac{-1}{8(x^2+3)^4} + k$

d) $\int \frac{1}{x} \ln^3 x \, dx = \frac{\ln^4 |x|}{4} + k$

Ejercicio 22:

Resuelve las siguientes integrales:

a) $\int x^4 \, e^{x^5} \, dx$ **b)** $\int x \, sen \, x^2 \, dx$ **c)** $\int \frac{dx}{\sqrt{9-x^2}}$ **d)** $\int \frac{x \, dx}{\sqrt{x^2+5}}$

a) $\int x^4 \, e^{x^5} \, dx = \frac{1}{5} \int 5x^4 \, e^{x^5} \, dx = \frac{1}{5} e^{x^5} + k$

b) $\int x \, sen \, x^2 \, dx = \frac{1}{2} \int 2x \, sen \, x^2 \, dx = \frac{-1}{2} \cos x^2 + k$

c) $\int \frac{dx}{\sqrt{9-x^2}} = \int \frac{1/3 \, dx}{\sqrt{1-(x/3)^2}} = arc \, sen \left(\frac{x}{3}\right) + k$

d) $\int \frac{x \, dx}{\sqrt{x^2+5}} = \sqrt{x^2+5} + k$

Ejercicio 23:

Resuelve las siguientes integrales:

a) $\int \sqrt{x^2-2x}\,(x-1)\,dx$ **b)** $\int tg \, x \, sec^2 x \, dx$

c) $\int \frac{(1+\ln x)^2}{x} \, dx$ **d)** $\int \sqrt{(1+\cos x)^3} \, sen \, x \, dx$

a) $\int \sqrt{x^2-2x}\,(x-1)\,dx = \frac{1}{2} \int \sqrt{x^2-2x}\,(2x-2)\,dx = \frac{1}{2} \int (x^2-2x)^{1/2}(2x-2)\,dx =$

$= \frac{1}{2} \frac{(x^2-2x)^{3/2}}{3/2} + k = \frac{\sqrt{(x^2-2x)^3}}{3} + k$

b) $\int tg \, x \, sec^2 x \, dx = \frac{tg^2 x}{2} + k$

c) $\int \frac{(1+\ln x)^2}{x} \, dx = \int (1+\ln x)^2 \cdot \frac{1}{x} \, dx = \frac{(1+\ln|x|)^3}{3} + k$

d) $\int \sqrt{(1+\cos x)^3} \, sen \, x \, dx = -\int (1+\cos x)^{3/2} (-sen \, x) \, dx = -\frac{(1+\cos x)^{5/2}}{5/2} + k =$

$= \frac{-2\sqrt{(1+\cos x)^5}}{5} + k$

Ejercicio 24:

Aplica la integración por partes para resolver las siguientes integrales:

a) $\int x \ln x\, dx$ b) $\int e^x \cos x\, dx$ c) $\int x^2\, sen\, x\, dx$ d) $\int x^2 e^{2x}\, dx$

e) $\int \cos(\ln x)\, dx$ f) $\int x^2 \ln x\, dx$ g) $\int arc\, tg\, x\, dx$ h) $\int (x+1)^2 e^x\, dx$

a) $\int x \ln x\, dx$

$$\begin{cases} u = \ln x \;\rightarrow\; du = \frac{1}{x}\, dx \\ dv = x\, dx \;\rightarrow\; v = \frac{x^2}{2} \end{cases}$$

$$\int x \ln x\, dx = \frac{x^2}{2} \ln x - \int \frac{x}{2}\, dx = \frac{x^2}{2} \ln|x| - \frac{x^2}{4} + k$$

b) $\int e^x \cos x\, dx$

$$\begin{cases} u = e^x \;\rightarrow\; du = e^x\, dx \\ dv = \cos x\, dx \;\rightarrow\; v = sen\, x \end{cases}$$

$$\int e^x \cos x\, dx = e^x\, sen\, x - \underbrace{\int e^x\, sen\, x\, dx}_{I_1}$$

$$\begin{cases} u_1 = e^x \;\rightarrow\; du_1 = e^x\, dx \\ dv_1 = sen\, x\, dx \;\rightarrow\; v_1 = -\cos x \end{cases}$$

$$I_1 = -e^x \cos x + \int e^x \cos x\, dx$$

Por tanto:

$$\int e^x \cos x\, dx = e^x\, sen\, x + e^x \cos x - \int e^x \cos x\, dx$$

$$2\int e^x \cos x\, dx = e^x\, sen\, x + e^x \cos x$$

$$\int e^x \cos x\, dx = \frac{e^x\, sen\, x + e^x \cos x}{2} + k$$

c) $\int x^2 \, sen \, x \, dx$

$$\begin{cases} u = x^2 & \rightarrow \quad du = 2x \, dx \\ dv = sen \, x \, dx & \rightarrow \quad v = -cos \, x \end{cases}$$

$$\int x^2 \, sen \, x \, dx = -x^2 \cos x + \int 2x \cos x \, dx = -x^2 \cos x + 2 \underbrace{\int x \cos x \, dx}_{I_1}$$

$$\begin{cases} u_1 = x & \rightarrow \quad du_1 = dx \\ dv_1 = cos \, x \, dx & \rightarrow \quad v_1 = sen \, x \end{cases}$$

$$I_1 = x \, sen \, x - \int sen \, x \, dx = x \, sen \, x + cos \, x$$

Por tanto:

$$\int x^2 \, sen \, x \, dx = -x^2 \cos x + 2x \, sen \, x + 2 \cos x + k$$

d) $\int x^2 e^{2x} \, dx$

$$\begin{cases} u = x^2 & \rightarrow \quad du = 2x \, dx \\ dv = e^{2x} \, dx & \rightarrow \quad v = \frac{1}{2} e^{2x} \end{cases}$$

$$\int x^2 e^{2x} \, dx = \frac{x^2}{2} e^{2x} - \underbrace{\int x \, e^{2x} \, dx}_{I_1}$$

$$\begin{cases} u_1 = x & \rightarrow \quad du_1 = dx \\ dv_1 = e^{2x} \, dx & \rightarrow \quad v_1 = \frac{1}{2} e^{2x} \end{cases}$$

$$I_1 = \frac{x}{2} e^{2x} - \int \frac{1}{2} e^{2x} \, dx = \frac{x}{2} e^{2x} - \frac{1}{4} e^{2x}$$

Por tanto: $\int x^2 e^{2x} \, dx = \frac{x^2}{2} e^{2x} - \frac{x}{2} e^{2x} + \frac{1}{4} e^{2x} + k = \left(\frac{x^2}{2} - \frac{x}{2} + \frac{1}{4} \right) e^{2x} + k$

e) $\int cos(ln\, x) dx$

$$\begin{cases} u = cos\,(ln\, x) \rightarrow du = -sen\,(ln\, x) \cdot \frac{1}{x}\, dx \\ dv = dx \rightarrow v = x \end{cases}$$

$$\int cos\,(ln\, x)\, dx = x\, cos\,(ln\, x) + \underbrace{\int sen\,(ln\, x)\, dx}_{I_1}$$

$$\begin{cases} u_1 = sen\,(ln\, x) \rightarrow du_1 = cos\,(ln\, x) \cdot \frac{1}{x}\, dx \\ dv_1 = dx \rightarrow v_1 = x \end{cases}$$

$$I_1 = x\, sen\,(ln\, x) - \int cos\,(ln\, x)\, dx$$

Por tanto:

$$\int cos\,(ln\, x)\, dx = x\, cos\,(ln\, x) + x\, sen\,(ln\, x) - \int cos\,(ln\, x)\, dx$$

$$2\int cos\,(ln\, x)\, dx = x\, cos\,(ln\, x) + x\, sen\,(ln\, x)$$

$$\int cos\,(ln\, x)\, dx = \frac{x\, cos\,(ln\, x) + x\, sen\,(ln\, x)}{2} + k$$

f) $\int x^2\, ln\, x\, dx$

$$\begin{cases} u = ln\, x \rightarrow du = \frac{1}{x}\, dx \\ dv = x^2\, dx \rightarrow v = \frac{x^3}{2} \end{cases}$$

$$\int x^2\, ln\, x\, dx = \frac{x^3\, ln\, x}{3} - \int \frac{x^2}{3}\, dx = \frac{x^3\, ln\, x}{3} - \frac{x^3}{9} + k$$

g) $\int arc\ tg\ x\ dx$

$$\begin{cases} u = arc\ tg\ x & \rightarrow & du = \dfrac{1}{1+x^2}\ dx \\ dv = dx & \rightarrow & v = x \end{cases}$$

$$\int arc\ tg\ x = x\ arc\ tg\ x - \int \frac{1}{1+x^2}\ dx = x\ arc\ tg\ x - \frac{1}{2}\int \frac{2x}{1+x^2}\ dx =$$

$$= x\ arc\ tg\ x - \frac{1}{2}\ ln\ (1+x^2) + k$$

h) $\int (x+1)^2\ e^x\ dx$

$$\begin{cases} u = (x+1)^2 & \rightarrow & du = 2(x+1)\ dx \\ dv = e^x\ dx & \rightarrow & v = e^x \end{cases}$$

$$\int (x+1)^2\ e^x\ dx = (x+1)^2\ e^x - 2\underbrace{\int (x+1)\ e^x\ dx}_{I_1}$$

$$\begin{cases} u_1 = (x+1) & \rightarrow & du_1 = dx \\ dv_1 = e^x\ dx & \rightarrow & v_1 = e^x \end{cases}$$

$$I_1 = (x+1)\ e^x - \int e^x\ dx = (x+1)\ e^x - e^x = (x+1-1)\ e^x = x\ e^x$$

Por tanto:

$$\int (x+1)^2\ e^x\ dx = (x+1)^2\ e^x - 2x\ e^x + k =$$

$$= (x^2 + 2x + 1 - 2x)\ e^x + k = (x^2+1)\ e^x + k$$

Ejercicio 25:

Determina el valor de las integrales que se proponen a continuación:

a) $\int x \cdot 2^{-x}\ dx$ **b)** $\int arc\ cos\ x\ dx$ **c)** $\int x\ cos\ 3x\ dx$ **d)** $\int x^5\ e^{-x^3}\ dx$

a) $\int x \cdot 2^{-x}\ dx$

$$\begin{cases} u = x & \rightarrow & du = dx \\ dv = 2^{-x}\ dx & \rightarrow & v = \dfrac{-2^{-x}}{ln\ 2} \end{cases}$$

$$\int x\ 2^{-x}\ dx = \frac{-x \cdot 2^{-x}}{ln\ 2} + \int \frac{2^{-x}}{ln\ 2}\ dx = \frac{-x \cdot 2^{-x}}{ln\ 2} + \frac{1}{ln\ 2}\int 2^{-x}\ dx =$$

$$= \frac{-x \cdot 2^{-x}}{ln\ 2} - \frac{2^{-x}}{(ln\ 2)^2} + k$$

b) $\int arc\ cos\ x\ dx$

$$\begin{cases} u = arc\ cos\ x & \rightarrow & du = \dfrac{-1}{\sqrt{1-x^2}}\ dx \\ dv = dx & \rightarrow & v = x \end{cases}$$

$$\int arc\ cos\ x\ dx = x\ arc\ cos\ x - \int \frac{-x}{\sqrt{1-x^2}}\ dx = x\ arc\ cos\ x - \sqrt{1-x^2} + k$$

c) $\int x \cos 3x \, dx$

$$\begin{cases} u = x \quad \rightarrow \quad du = dx \\ dv = \cos 3x \, dx \quad \rightarrow \quad v = \frac{1}{3} \operatorname{sen} 3x \end{cases}$$

$$\int x \cos 3x \, dx = \frac{x}{3} \operatorname{sen} 3x - \frac{1}{3} \int \operatorname{sen} 3x \, dx = \frac{x}{3} \operatorname{sen} 3x + \frac{1}{9} \cos 3x + k$$

d) $\int x^5 e^{-x^3} dx = \int \underbrace{x^3}_{u} \cdot \underbrace{x^2 e^{-x^3} dx}_{dv}$

$$\begin{cases} u = x^3 \quad \rightarrow \quad du = 3x^2 \, dx \\ dv = x^2 e^{-x^3} dx \quad \rightarrow \quad v = \frac{-1}{3} e^{-x^3} \end{cases}$$

$$\int x^5 e^{-x^3} dx = \frac{-x^3}{3} e^{-x^3} + \int x^2 e^{-x^3} dx = \frac{-x^3}{3} e^{-x^3} - \frac{1}{3} e^{-x^3} + k =$$

$$= \frac{(-x^3 - 1)}{3} e^{-x^3} + k$$

Ejercicio 26:

Resuelve las siguientes integrales:

a) $\int \frac{2x - 4}{(x - 1)^2 (x + 3)} dx$

b) $\int \frac{2x + 3}{(x - 2)(x + 5)} dx$

c) $\int \frac{1}{(x - 1)(x + 3)^2} dx$

d) $\int \frac{3x - 2}{x^2 - 4} dx$

a) $\int \frac{2x - 4}{(x - 1)^2 (x + 3)} dx$

Descomponemos en fracciones simples:

$$\frac{2x - 4}{(x - 1)^2 (x + 3)} = \frac{A}{x - 1} + \frac{B}{(x - 1)^2} + \frac{C}{x + 3}$$

$$\frac{2x - 4}{(x - 1)^2 (x + 3)} = \frac{A(x - 1)(x + 3) + B(x + 3) + C(x - 1)^2}{(x - 1)^2 (x + 3)}$$

$$2x - 4 = A(x - 1)(x + 3) + B(x + 3) + C(x - 1)^2$$

Hallamos A, B y C:

$$\left.\begin{array}{llll} x = 1 & \rightarrow & -2 = 4B & \rightarrow \quad B = -1/2 \\ x = -3 & \rightarrow & -10 = 16C & \rightarrow \quad C = -5/8 \\ x = 0 & \rightarrow & -4 = -3A + 3B + C & \rightarrow \quad A = 5/8 \end{array}\right\}$$

Por tanto:

$$\int \frac{2x-4}{(x-1)^2\,(x+3)}\,dx = \int \frac{5/8}{x-1}\,dx + \int \frac{-1/2}{(x-1)^2}\,dx + \int \frac{-5/8}{x+3}\,dx =$$

$$= \frac{5}{8}\,ln|x-1| + \frac{1}{2}\cdot\frac{1}{(x-1)} - \frac{5}{8}\,ln|x+3| + k = \frac{5}{8}\,ln\left|\frac{x-1}{x+3}\right| + \frac{1}{2x-2} + k$$

b) $\int \frac{2x+3}{(x-2)\,(x+5)}\,dx$

Descomponemos en fracciones simples:

$$\frac{2x+3}{(x-2)\,(x+5)} = \frac{A}{x-2} + \frac{B}{x+5} = \frac{A(x+5)+B(x-2)}{(x-2)\,(x+5)}$$

$$2x+3 = A(x+5)+B(x-2)$$

Hallamos A y B:

$$\left.\begin{array}{lllll} x=2 & \to & 7=7A & \to & A=1 \\ x=-5 & \to & -7=-7B & \to & B=1 \end{array}\right\}$$

Por tanto:

$$\int \frac{2x+3}{(x-2)\,(x+5)}\,dx = \int \frac{1}{x-2}\,dx + \int \frac{1}{x+5}\,dx =$$

$$= ln|x-2| + ln|x+5| + k = ln|(x-2)(x+5)| + k$$

c) $\int \frac{1}{(x-1)\,(x+3)^2}\,dx$

Descomponemos en fracciones simples:

$$\frac{1}{(x-1)\,(x+3)^2} = \frac{A}{x-1} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$$

$$\frac{1}{(x-1)\,(x+3)^2} = \frac{A(x+3)^2 + B(x-1)(x+3) + C(x-1)}{(x-1)\,(x+3)^2}$$

$$1 = A(x+3)^2 + B(x-1)(x+3) + C(x-1)$$

Hallamos A, B y C:

$$\left.\begin{array}{lllll} x=1 & \to & 1=16A & \to & A=1/16 \\ x=-3 & \to & 1=-4C & \to & C=-1/4 \\ x=0 & \to & 1=9A-3B-C & \to & B=-1/16 \end{array}\right\}$$

Por tanto:

$$\int \frac{1}{(x-1)\,(x+3)^2}\,dx = \int \frac{1/16}{x-1}\,dx + \int \frac{-1/16}{x+3}\,dx + \int \frac{-1/4}{(x+3)^2}\,dx =$$

$$= \frac{1}{16}\,ln|x-1| - \frac{1}{16}\,ln|x+3| + \frac{1}{4}\cdot\frac{1}{(x+3)} + k =$$

$$= \frac{1}{16}\,ln\left|\frac{x-1}{x+3}\right| + \frac{1}{4(x+3)} + k$$

d) $\int \frac{3x-2}{x^2-4}\,dx = \int \frac{3x-2}{(x-2)(x+2)}\,dx$

Descomponemos en fracciones simples:

$$\frac{3x-2}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2} = \frac{A(x+2)+B(x-2)}{(x-2)(x+2)}$$

$$3x-2 = A(x+2) + B(x-2)$$

Hallamos A y B:

$$\left.\begin{array}{lllll} x=2 & \rightarrow & 4=4A & \rightarrow & A=1 \\ x=-2 & \rightarrow & -8=-4B & \rightarrow & B=2 \end{array}\right\}$$

Por tanto:

$$\int \frac{3x-2}{x^2-4}\,dx = \int \frac{1}{x-2}\,dx + \int \frac{2}{x+2}\,dx =$$

$$= ln|x-2| + 2\,ln|x+2| + k = ln\left[|x-2|(x+2)^2\right] + k$$

Ejercicio 27:

Resuelve las integrales:

a) $\int \frac{ln\,x}{x}\,dx$ **b)** $\int \frac{1-sen\,x}{x+cos\,x}\,dx$ **c)** $\int \frac{1}{x\,ln\,x}\,dx$

d) $\int \frac{1+e^x}{e^x+x}\,dx$ **e)** $\int \frac{sen\,(1/x)}{x^2}\,dx$ **f)** $\int \frac{2x-3}{x+2}\,dx$

g) $\int \frac{arc\,tg\,x}{1+x^2}\,dx$ **h)** $\int \frac{sen\,x}{cos^4\,x}\,dx$

a) $\int \frac{ln\,x}{x}\,dx = \int \frac{1}{x}\,ln\,x\,dx = \frac{ln^2|x|}{2} + k$

b) $\int \frac{1-sen\,x}{x+cos\,x}\,dx = ln|x+cos\,x| + k$

c) $\int \frac{1}{x\,ln\,x}\,dx = \int \frac{1/x}{ln\,x}\,dx = ln|ln|x|| + k$

d) $\int \frac{1+e^x}{e^x+x}\,dx = ln|e^x+x| + k$

e) $\int \frac{sen\,(1/x)}{x^2}\,dx = -\int \frac{-1}{x^2}\,sen\left(\frac{1}{x}\right)dx = cos\left(\frac{1}{x}\right) + k$

f) $\int \frac{2x-3}{x+2}\,dx = \int \left(2 - \frac{7}{x+2}\right)dx = 2x - 7\,ln|x+2| + k$

g) $\int \frac{arc\ tg\ x}{1+x^2}\,dx = \int \frac{1}{1+x^2}\ arc\ tg\ x\,dx = \frac{arc\ tg^2\ x}{2} + k$

h) $\int \frac{sen\ x}{cos^4\ x}\,dx = -\int (-sen\ x)(cos\ x)^{-4}\,dx = \frac{-(cos\ x)^{-3}}{-3} + k = \frac{1}{3\ cos^3\ x} + k$

Ejercicio 28:

Resuelve por sustitución:

a) $\int x\sqrt{x+1}\,dx$ **b)** $\int \frac{dx}{x-\sqrt[4]{x}}$ **c)** $\int \frac{x}{\sqrt{x+1}}\,dx$

d) $\int \frac{1}{x\sqrt{x+1}}\,dx$ **e)** $\int \frac{1}{x+\sqrt{x}}\,dx$ **f)** $\int \frac{\sqrt{x}}{1+x}\,dx$

a) Haz $x+1=t^2$. ***b) Haz*** $x=t^4$.

a) $\int x\sqrt{x+1}\,dx$

Cambio: $x+2=t^2 \;\rightarrow\; dx=2t\,dt$

$$\int x\sqrt{x+1}\,dx = \int (t^2-1)t\cdot 2t\,dt = \int (2t^4-2t^2)\,dt = \frac{2t^5}{5} - \frac{2t^3}{3} + k =$$

$$= \frac{2\sqrt{(x+1)^5}}{5} - \frac{2\sqrt{(x+1)^3}}{3} + k$$

b) $\int \frac{dx}{x-\sqrt[4]{x}}$

Cambio: $x=t^4 \;\rightarrow\; dx=4t^3\,dt$

$$\int \frac{dx}{x-\sqrt[4]{x}} = \int \frac{4t^3\,dt}{t^4-t} = \int \frac{4t^2\,dt}{t^3-1} = \frac{4}{3}\int \frac{3t^2\,dt}{t^3-1} = \frac{4}{3}\,ln\,|t^3-1| + k =$$

$$= \frac{4}{3}\,ln\,|\sqrt[4]{x^3}-1| + k$$

c) $\int \frac{x}{\sqrt{x+1}}\,dx$

Cambio: $x+1=t^2 \;\rightarrow\; dx=2t\,dt$

$$\int \frac{x}{\sqrt{x+1}}\,dx = \int \frac{(t^2-1)}{t}\cdot 2t\,dt = \int (2t^2-2)\,dt = \frac{2t^3}{3} - 2t + k =$$

$$= \frac{2\sqrt{(x+1)^3}}{3} - 2\sqrt{x+1} + k$$

d) $\displaystyle\int \frac{1}{x\sqrt{x+1}}\,dx$

Cambio: $x + 1 = t^2 \;\rightarrow\; dx = 2t\,dt$

$$\int \frac{1}{x\sqrt{x+1}}\,dx = \int \frac{2t\,dt}{(t^2-1)t} = \int \frac{2\,dt}{(t+1)(t-1)}$$

Descomponemos en fracciones simples:

$$\frac{2}{(t+1)(t-1)} = \frac{A}{t+1} + \frac{B}{t-1} = \frac{A(t-1)+B(t+1)}{(t+1)(t-1)}$$

$2 = A(t-1) + B(t+1)$

Hallamos A y B:

$$\left.\begin{array}{lll} t=-1 & \rightarrow\; 2=-2A & \rightarrow\; A=-1 \\ t=1 & \rightarrow\; 2=2B & \rightarrow\; B=1 \end{array}\right\}$$

Por tanto:

$$\int \frac{2\,dt}{(t+1)(t-1)} = \int \left(\frac{-1}{t+1} + \frac{1}{t-1}\right) dt = -ln\,|t+1| + ln\,|t-1| + k =$$

$$= ln\left|\frac{t-1}{t+1}\right| + k$$

Así:

$$\int \frac{1}{x\sqrt{x+1}}\,dx = ln\left|\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right| + k$$

e) $\displaystyle\int \frac{1}{x+\sqrt{x}}\,dx$

Cambio: $x = t^2 \;\rightarrow\; dx = 2t\,dt$

$$\int \frac{1}{x+\sqrt{x}}\,dx = \int \frac{2t\,dt}{t^2+t} = \int \frac{2\,dt}{t+1} = 2\,ln\,|t+1| + k =$$

$$= 2\,ln\,(\sqrt{x}+1) + k$$

f) $\displaystyle\int \frac{\sqrt{x}}{1+x}\,dx$

Cambio: $x = t^2 \;\rightarrow\; dx = 2t\,dt$

$$\int \frac{\sqrt{x}}{1+x}\,dx = \int \frac{t\cdot 2t\,dt}{1+t^2} = \int \frac{2t^2\,dt}{1+t^2} = \int \left(2 - \frac{2}{1+t^2}\right) dt =$$

$$= 2t - 2\,arc\,tg\,t + k = 2\sqrt{x} - 2\,arc\,tg\,\sqrt{x} + k$$

Ejercicio 29:

Resuelve, utilizando un cambio de variable, estas integrales:

a) $\int \sqrt{9-4x^2}\,dx$ **b)** $\int \frac{dx}{e^{2x}-3e^x}$ **c)** $\int \frac{e^{3x}-e^x}{e^{2x}+1}\,dx$ **d)** $\int \frac{1}{1+\sqrt{x}}\,dx$

☛ ***a) Haz*** $sen\, t = 2x/3$.

a) $\int \sqrt{9-4x^2}\,dx$

Cambio: $sen\, t = \frac{2x}{3} \rightarrow x = \frac{3}{2}\,sen\, t \rightarrow dx = \frac{3}{2}\cos t\, dt$

$$\int \sqrt{9-4x^2}\,dx = \int \sqrt{9-4\cdot\frac{9}{4}\,sen^2\, t}\cdot\frac{3}{2}\cos t\, dt = \int 3\cos t\cdot\frac{3}{2}\cos t\, dt =$$

$$= \frac{9}{2}\int \cos^2 t\, dt = \frac{9}{2}\int\left(\frac{1}{2}-\frac{\cos 2t}{2}\right)dt = \frac{9}{2}\left(\frac{1}{2}t+\frac{1}{4}\,sen\, 2t\right)+k =$$

$$= \frac{9}{4}t+\frac{9}{8}\,sen\, 2t + k = \frac{9}{4}\,arc\, sen\left(\frac{2x}{3}\right)+\frac{9}{8}\cdot 2\,sen\, t\cos t + k =$$

$$= \frac{9}{4}\,arc\, sen\left(\frac{2x}{3}\right)+\frac{9}{4}\cdot\frac{2x}{3}\sqrt{1-\frac{4x^2}{9}}+k =$$

$$= \frac{9}{4}\,arc\, sen\left(\frac{2x}{3}\right)+\frac{x}{2}\cdot\sqrt{9-4x^2}+k$$

b) $\int \frac{dx}{e^{2x}-3e^x}$

Cambio: $e^x = t \rightarrow x = \ln t \rightarrow dx = \frac{1}{t}\,dt$

$$\int \frac{dx}{e^{2x}-3e^x} = \int \frac{1/t}{t^2-3t}\,dt = \int \frac{1}{t^3-3t^2}\,dt = \int \frac{1}{t^2(t-3)}\,dt$$

Descomponemos en fracciones simples:

$$\frac{1}{t^2(t-3)} = \frac{A}{t}+\frac{B}{t^2}+\frac{C}{t-3} = \frac{At(t-3)+B(t-3)+Ct^2}{t^2(t-3)}$$

$$1 = At(t-3)+B(t-3)+Ct^2$$

Hallamos A, B y C:

$$\left.\begin{array}{lll} t = 0 & \rightarrow \quad 1 = -3B & \rightarrow \quad B = -1/3 \\ t = 3 & \rightarrow \quad 1 = 9C & \rightarrow \quad C = 1/9 \\ t = 1 & \rightarrow \quad 1 = -2A - 2B + C & \rightarrow \quad A = -1/9 \end{array}\right\}$$

Así, tenemos que:

$$\int \frac{1}{t^2(t-3)}\,dt = \int \left(\frac{-1/9}{t} + \frac{-1/3}{t^2} + \frac{1/9}{t-3} \right) dt =$$

$$= \frac{-1}{9}\,ln|t| + \frac{1}{3t} + \frac{1}{9}\,ln|t-3| + k$$

Por tanto:

$$\int \frac{dx}{e^{2x} - 3e^x} = \frac{-1}{9}\,ln\,e^x + \frac{1}{3e^x} + \frac{1}{9}\,ln|e^x - 3| + k =$$

$$= -\frac{1}{9}x + \frac{1}{3e^x} + \frac{1}{9}\,ln|e^x - 3| + k$$

c) $\displaystyle\int \frac{e^{3x} - e^x}{e^{2x} + 1}\,dx$

Cambio: $e^x = t \rightarrow x = ln\,t \rightarrow dx = \frac{1}{t}\,dt$

$$\int \frac{e^{3x} - e^x}{e^{2x} + 1}\,dx = \int \frac{t^3 - t}{t^2 + 1} \cdot \frac{1}{t}\,dt = \int \frac{t^2 - 1}{t^2 + 1}\,dt = \int \left(1 - \frac{2}{t^2 + 1} \right) dt =$$

$$= t - 2\,arc\,tg\,t + k = e^x - 2\,arc\,tg\,(e^x) + k$$

d) $\displaystyle\int \frac{1}{1 + \sqrt{x}}\,dx$

Cambio: $x = t^2 \rightarrow dx = 2t\,dt$

$$\int \frac{1}{1 + \sqrt{x}}\,dx = \int \frac{2t\,dt}{1 + t} = \int \left(2 - \frac{2}{1 + t} \right) dt = 2t - 2\,ln|1 + t| + k =$$

$$= 2\sqrt{x} - 2\,ln\,(1 + \sqrt{x}) + k$$

Ejercicio 30:

Encuentra la primitiva de $f(x) = \dfrac{1}{1 + 3x}$ que se anula para $x = 0$.

$$F(x) = \int \frac{1}{1 + 3x}\,dx = \frac{1}{3}\int \frac{3}{1 + 3x}\,dx = \frac{1}{3}\,ln|1 + 3x| + k$$

$$F(0) = k = 0$$

Por tanto: $F(x) = \dfrac{-1}{3}\,ln|1 + 3x|$

Ejercicio 31:

Halla la función F para la que $F'(x) = \frac{1}{x^2}$ y $F(1) = 2$.

$$F(x) = \int \frac{1}{x^2}\, dx = \frac{-1}{x} + k$$

$$F(1) = -1 + k = 2 \quad \Rightarrow \quad k = 3$$

Por tanto: $F(x) = \frac{-1}{x} + 3$

Ejercicio 32:

De todas las primitivas de la función $y = 4x - 6$, ¿cuál de ellas toma el valor 4 para $x = 1$?

$$F(x) = \int (4x - 6)\, dx = 2x^2 - 6x + k$$

$$F(1) = 2 - 6 + k = 4 \quad \Rightarrow \quad k = 8$$

Por tanto: $F(x) = 2x^2 - 6x + 8$

Ejercicio 33:

Halla $f(x)$ sabiendo que $f''(x) = 6x$, $f'(0) = 1$ y $f(2) = 5$.

$$\left.\begin{array}{l} f'(x) = \int 6x\, dx = 3x^2 + c \\ f'(0) = c = 1 \end{array}\right\} \quad f'(x) = 3x^2 + 1$$

$$\left.\begin{array}{l} f(x) = \int (3x^2 + 1)\, dx = x^3 + x + k \\ f(2) = 10 + k = 5 \quad \Rightarrow \quad k = -5 \end{array}\right\}$$

Por tanto: $f(x) = x^3 + x - 5$

Ejercicio 34:

Resuelve las siguientes integrales por sustitución:

a) $\int \frac{e^x}{1 - \sqrt{e^x}}\, dx$ **b)** $\int \sqrt{e^x - 1}\, dx$

☛ ***a) Haz $\sqrt{e^x} = t$. b) Haz $\sqrt{e^x - 1} = t$.***

a) $\int \frac{e^x}{1 - \sqrt{e^x}}\, dx$

Cambio: $\sqrt{e^x} = t \quad \rightarrow \quad e^{x/2} = t \quad \rightarrow \quad \frac{x}{2} = \ln t \quad \rightarrow \quad dx = \frac{2}{t}\, dt$

$$\int \frac{e^x}{1 - \sqrt{e^x}} = \int \frac{t^2 \cdot (2/t)\, dt}{1 - t} = \int \frac{2t\, dt}{1 - t} = \int \left(-2 + \frac{2}{1 - t}\right) dt =$$

$$= -2t - 2 \ln|1 - t| + k = -2\sqrt{e^x} - 2 \ln|1 - \sqrt{e^x}| + k$$

b) $\int \sqrt{e^x - 1}\; dx$

$$Cambio: \sqrt{e^x - 1} = t \;\rightarrow\; e^x = t^2 + 1 \;\rightarrow\; x = ln\,(t^2 + 1) \;\rightarrow\; dx = \frac{2t}{t^2+1}\,dt$$

$$\int \sqrt{e^x - 1}\; dx = \int t \cdot \frac{2t}{t^2+1}\,dt = \int \frac{2t^2}{t^2+1}\,dt = \int \left(2 - \frac{2}{t^2+1}\right) dt =$$

$$= 2t - 2\,arc\,tg\,t + k = 2\sqrt{e^x - 1} - 2\,arc\,tg\,\sqrt{e^x - 1} + k$$

Ejercicio 35:

Determina la función $f(x)$ sabiendo que:

$$f''(x) = x\,ln\,x,\; f'(1) = 0 \;\text{ y }\; f(e) = \frac{e}{4}$$

$$f'(x) = \int x\,ln\,x\,dx$$

Integramos por partes:

$$\begin{cases} u = ln\,x \;\rightarrow\; du = \dfrac{1}{x}\,dx \\ dv = x\,dx \;\rightarrow\; v = \dfrac{x^2}{2} \end{cases}$$

$$\left.\begin{array}{l} f'(x) = \dfrac{x^2}{2}\,ln\,x - \displaystyle\int \dfrac{x}{2}\,dx = \dfrac{x^2}{2}\,ln\,x - \dfrac{x^2}{4} + k = \dfrac{x^2}{2}\left(ln\,x - \dfrac{1}{2}\right) + k \\ f'(1) = \dfrac{1}{2}\left(-\dfrac{1}{2}\right) + k = -\dfrac{1}{4} + k = 0 \;\Rightarrow\; k = \dfrac{1}{4} \end{array}\right\}$$

$$f'(x) = \frac{x^2}{2}\left(ln\,x - \frac{1}{2}\right) + \frac{1}{4}$$

$$f(x) = \int \left[\frac{x^2}{2}\left(ln\,x - \frac{1}{2}\right) + \frac{1}{4}\right] dx = \underbrace{\int \frac{x^2}{2}\left(ln\,x - \frac{1}{2}\right) dx}_{I} + \frac{1}{4}\,x$$

$$\begin{cases} u = \left(ln\,x - \dfrac{1}{2}\right) \;\rightarrow\; du = \dfrac{1}{x}\,dx \\ dv = \dfrac{x^2}{2}\,dx \;\rightarrow\; v = \dfrac{x^3}{6} \end{cases}$$

$$I = \frac{x^3}{6}\left(ln\,x - \frac{1}{2}\right) - \int \frac{x^2}{6}\,dx = \frac{x^3}{6}\left(ln\,x - \frac{1}{2}\right) - \frac{x^3}{18} + k$$

Por tanto:

$$\left.\begin{array}{l} f(x) = \dfrac{x^3}{6}\left(ln\ x - \dfrac{1}{2}\right) - \dfrac{x^3}{18} + \dfrac{1}{4}x + k \\ f(e) = \dfrac{e^3}{12} - \dfrac{e^3}{18} + \dfrac{e}{4} + k = \dfrac{e^3}{36} + \dfrac{e}{4} + k = \dfrac{e}{4} \quad \Rightarrow \quad k = -\dfrac{e^3}{36} \end{array}\right\}$$

$$f(x) = \frac{x^3}{6}\left(ln\ x - \frac{1}{2}\right) - \frac{x^3}{18} + \frac{1}{4}x - \frac{e^3}{36}$$

Ejercicio 36:

Calcula la expresión de una función $f(x)$ tal que $f'(x) = x\, e^{-x^2}$ y que $f(0) = \dfrac{1}{2}$.

$$f(x) = \int x\, e^{-x^2}\, dx = -\frac{1}{2}\int -2x\, e^{-x^2}\, dx = -\frac{1}{2}\, e^{-x^2} + k$$

$$f(0) = -\frac{1}{2} + k = \frac{1}{2} \quad \Rightarrow \quad k = 1$$

Por tanto: $f(x) = -\dfrac{1}{2}\, e^{-x^2} + 1$

Ejercicio 37:

Encuentra la función derivable f: $[-1, 1] \to \mathbb{R}$ que cumple $f(1) = -1$ y

$$f'(x) = \begin{cases} x^2 - 2x & \text{si } -1 \le x < 0 \\ e^x - 1 & \text{si } 0 \le x \le 1 \end{cases}$$

- Si $x \neq 0$:

$$f(x) = \begin{cases} \dfrac{x^3}{3} - x^2 + k & \text{si } -1 \le x < 0 \\ e^x - x + c & \text{si } 0 < x \le 1 \end{cases}$$

- Hallamos k y c teniendo en cuenta que $f(1) = -1$ y que $f(x)$ ha de ser continua en $x = 0$.

$$f(1) = -1 \quad \Rightarrow \quad e - 1 + c = -1 \quad \Rightarrow \quad c = -e$$

$$\left.\begin{array}{l} \lim\limits_{x \to 0^-} f(x) = k \\ \lim\limits_{x \to 0^+} f(x) = 1 - e \end{array}\right\} k = 1 - e$$

Por tanto: $f(x) = \begin{cases} \dfrac{x^3}{3} - x^2 + 1 - e & \text{si } -1 \le x < 0 \\ e^x - x - e & \text{si } 0 \le x \le 1 \end{cases}$

Ejercicio 38:

De una función derivable se sabe que pasa por el punto $A(-1,-4)$ y que su derivada es:

$$f'(x) = \begin{cases} 2-x & \text{si } x \le 1 \\ 1/x & \text{si } x > 1 \end{cases}$$

a) Halla la expresión de $f(x)$.

b) Obtén la ecuación de la recta tangente a $f(x)$ en $x = 2$.

a) Si $x \neq 1$:

$$f(x) = \begin{cases} 2x - \dfrac{x^2}{2} + k & \text{si } x < 1 \\ \ln x + c & \text{si } x > 1 \end{cases}$$

Hallamos k y c teniendo en cuenta que $f(-1) = -4$ y que $f(x)$ ha de ser con tinua en $x = 1$.

$$f(-1) = -\frac{5}{2} + k = -4 \Rightarrow k = -\frac{3}{2}$$

$$\left.\begin{aligned} &\lim_{x \to 1^-} f(x) = \frac{3}{2} - \frac{3}{2} = 0 \\ &\lim_{x \to 1^+} f(x) = c \end{aligned}\right\} c = 0$$

Por tanto: $f(x) = \begin{cases} 2x - \dfrac{x^2}{2} - \dfrac{3}{2} & \text{si } x < 1 \\ \ln x & \text{si } x \ge 1 \end{cases}$

b) $f(2) = \ln 2$; $f'(2) = \dfrac{1}{2}$

La ecuación de la recta tangente será: $y = \ln 2 + \dfrac{1}{2}(x-2)$